A Novel Fuzzy Multiple Reference Model Adaptive Controller Design

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Abstract

This paper presents a novel Fuzzy Multiple Reference Model Adaptive Controller (FMRMAC) for systems that are multi modal in nature and show scheduled 'Jumps' in their operation. The proposed scheme incorporates a fuzzy logic switching method in the Model Reference Adaptive Control (MRAC) framework without any explicit identifier. The fuzzy switching scheme produces a 'soft' way of model generation, combining a group of weighted reference models effective at each modal operation. This approach can be performed online and is very well suited for applications that show sudden operational changes. Unlike static multiple model algorithms, or switching dynamic algorithms, this scheme provides interactive multiple model environment with soft switching. The scheme is effective, computationally feasible and fault tolerant in nature.

Keywords: Fuzzy Multiple Reference Model Adaptive Controller, Scheduled 'Jumps', Multi-Modal Systems.

1. Introduction

Control of Multimodal systems can be divided as temporal multimodal systems and spatial systems [1] and Multiple Model Adaptive Control (MMAC) is an effective solution for controlling such systems [2]. In general, stochastic and deterministic based mathematical MMAC designs [3]-[10] result in fairly complex algorithms, especially in the digital domain. Moreover, the application of these algorithms is normally carried out on linearized models at different operating points of nonlinear systems. On the other hand, heuristic based MMAC approaches using fuzzy logic schemes [11]-[13] deals with explicit identification of the plant and in turn generating the models, which involves the process of performance index calculation to choose the best ones.

Considering the current research status in this area, following observations can be made. First, heuristic based MMAC algorithms have definite advantages as opposed to conventional ones. Nevertheless, almost all the work done till now uses fuzzy logic schemes for modeling a system and then controlling it either by traditional or fuzzy structure. This creates the possibility of developing heuristic solutions and not optimum results. A combination of heuristic switching algorithm with optimal adaptive controller is proposed in [14] which has been applied to control electric machine and robotic manipulator. Though this scheme is very effective, the switching scheme has been performed at predefined time interval.

An entirely new approach is developed in this paper with the concept of generating a fuzzified reference model structure based on two important methods. To this end, an Intelligent Supervisory Loop (ISL) is incorporated into the traditional MRAC framework by utilizing a fuzzy logic switching scheme in order to generate the plant reference model at every control interval. The technique has been named as Fuzzy Multiple Reference Model Adaptive Controller (FMRMAC). Note that, unlike modeling the system and creating multiple models, a structural change in the reference model without implicit identification is the main focus. The main and original contributions of this approach are as follows. First, the controller is adaptive which implicitly identifies system mode changes that is every effective as opposed to heuristic techniques. Secondly, unlike in MMAC, the system models are not monitored to switch the controller but a unique changing fuzzified reference model is used so that one controller is effective for all system modes. This is extremely important from the viewpoint of alleviating serious issues related to on line tuning and switching. Finally unlike other techniques [13], the approach uses stable optimal control law for plants operating in multiple modes. It is worth noting that other heuristic techniques including [13] does not utilize adaptive control law for multiple reference model generation (instead uses implicit system model development for one plant model of operation) making them unsuitable for comparison.

The paper is organized as follows: In section 2, the problem statement is presented. Section 3 discusses the concept of fuzzy multiple reference model generation. Section 4 details the fuzzy logic scheme design and section 5 discusses the adaptive control laws and overall stability. In section 6, simulation application example is discussed, followed by conclusions in section 7.

2. Design Concept and Problem Statement

The theme behind the approach is as follows. If the system under consideration can be modeled using one

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reference model, then the adaptive controller can be utilized for controlling the system to track the desired reference model output. However, if the reference model is rigid and the system behavior is away from the behavior of the reference model then the controller will be stressed and finally fail to control the plant. The main purpose of the intelligent module is to alleviate this rigidness in reference model. The intelligent module uses the auxiliary measurements/states and generates an appropriate reference model at each control interval, thereby provides a moving reference structure with respect to the plant.

The system representation and problem details are as follows. Assuming the system input is U and output y_i, the objective is to make the control error e_c (y_{mi}-y_i) tends to zero where y_{mi} is the output of the reference model at a specific mode. Representing in state space form,

$$y_i(t) = A_{m(t)} X(t) + b_{m(t)} U(t) \quad (1)$$

where $y_i(t)$ is the plant output at a specific mode, U(t) is the control input, X(t) is the state vector $[X_1(t)\ldots X_n(t)]^T \in R^n$, $A_i(t)=[a_{1i},a_{2i},\ldots,a_{ni}]^T \in R^{nxn}$ and b_i takes values from the set of H constant elements which represents the known modes as indexed by subscripts i where i $\in$ {1,2,…,H}.

Thus the parameter vector can be represented by the triple $\{(A_i,b_i,c_i),\ldots,(A_H,b_H,c_H)\}$ which changes its values depending on the modes of operation. Let the above set denote scheduled jump parameter for each mode specified by the parameter index i. The mode variable m(t) takes the form mapped into any of the values in the domain i $\in$ {1,2,…,H} and correspondingly $A_m(t)$ and $b_m(t)$ are time varying. The mapping is denoted by m(t) = γ[X(t-d)] where d represents the time delay. The system is now a spatial multimodal one because the dynamics are scheduled through the states mapping γ.

3. Multiple Fuzzy Reference Model Generation

Consider a fuzzy system output denoted by a function $f(\Omega)$ represented as

$$f(\Omega) = \frac{\sum_{i=1}^{r} r_i \mu_i}{\sum_{i=1}^{r} \mu_i} \quad (2)$$

The fuzzy system has r rules and μ_i is the membership function of the antecedent of i[th] rule given by the input Ω Where: Ω is a vector containing the relevant auxiliary states, $\Omega \in \Re^m$.

Assuming this fuzzy system is constructed in such a way that the $\sum_{i=1}^{r} \mu_i \neq 0$ for all relevant auxiliary states and the parameter r_i is the consequent of i[th] rule. Then

$$f(\Omega) = \frac{\sum_{i=1}^{r} r_i \mu_i}{\sum_{i=1}^{r} \mu_i} = M^T P \vartheta = \Phi * \vartheta \quad (3)$$

where:

$$M = \begin{bmatrix} 1/\chi_1(\Omega) \\ 1/\chi_2(\Omega) \\ \vdots \\ 1/\chi_{m-1}(\Omega) \end{bmatrix}, \quad \Omega \in \Re^m, \Phi = M^T P$$

$$P = \begin{bmatrix} p_{1,0} & \cdots & p_{r,0} \\ \vdots & \vdots & \vdots \\ p_{1,m-1} & \cdots & p_{n,m-1} \end{bmatrix} \text{ and } \vartheta = \frac{[\mu_1, \mu_2, \ldots, \mu_r]^T}{\sum_{i=1}^{r} \mu_i}.$$

Thus the function approximation by fuzzy scheme is equal to the product of a parameter vector Φ and weight matrix ϑ. The reference model is represented as

$$\dot{X}_m(t) = Am_{m(t)} X_m(t) + bm_{m(t)} S(t) \quad (4)$$

$$y_{mi}(t) = C^T X_m(t)$$

where $y_{mi}(t)$ is the reference model output, S(t) is the command signal $Am_{m(t)}$, $bm_{m(t)}$ are model parameters at mode m(t) and $Xm_{m(t)}$ is the state space vector.

The transfer function form will then be

$$W_{mi}(s) = y_{mi}(t)/S(t) = K_{mi} * Z_{mi} / R_{mi} \quad (5)$$

where K_{mi} is gain matrix, Z_{mi} is the zeros matrix with suitable locations in the system domain, R_{mi} is monic and Hurwitz in nature.

From (5), it can be seen that the numerator and the denominator are functions of state variables X and the location of the poles and zeros are further influenced by modes of operation of the plant. In order to include these modal transitions, (5) has to be combined with (3). This can be rewritten in the form of an observed value as

$$\hat{W}_{mi}(s) = f(\Omega) * (K_{mi} * Z_{mi} / R_{mi}) \quad (6)$$

Thus the changes in the system dynamics can be mapped through auxiliary states to changes in system polynomial roots or the poles/zero combination as,

$$\hat{y}_{mi}(t) = \hat{W}_{mi} * S(t) \underline{\underline{\Delta}} (\Phi_i * \vartheta_i) * W_{mi} * S(t) \quad (7)$$

For a constant command signal, the observed reference model output will be

$$\hat{y}_{mi}(t) = \hat{\nu}(\Phi_i, \vartheta_i, W_{mi}) \quad (8)$$

where $W_{mi}(t)$ is the estimate of $\hat{W}_{mi}(t)$, $\hat{y}_{mi}(t)$ is the observed reference model output for the i[th] mode, Φ_i is the parameter vector developed by the fuzzy system depending on operating points, ϑ_i is the membership function weights and W_{mi} is the corresponding reference model transfer function.

The membership function weights act as a performance index function in modifying the reference model output. Based on each modal transition of the system, the parameter vector Φ, ϑ changes. Subsequently the reference model output moves such that the closed loop system provides a stable output with the roots on the left half s plane. This movement in the reference model secures the system from becoming unstable. Moreover, the modal transitions are smooth in nature, which reduce the transients during the changes in system mode. More importantly this avoids any pre-defined calculation.

4. Design of Fuzzy Reference Model Generator

Two ways of designing the fuzzy logic scheme based on the two different methodologies to develop stable desired closed loop system trajectories is proposed. The fact that the closed loop system trajectories pattern changes with change in the plant operating modes gives us a relation between desired trajectory and plant operating mode.

a) Method 1

In this method, fuzzy logic rules are developed by changing the natural frequency of the reference model in such a way that system closed loop performance at each mode is stable. This will generate dominant poles and zeros of system transfer function at each operating point. Consider the reference model transfer function as in (9), which shows the required system response.

$$W_{mi}(s) = \frac{G(s)}{1+G(s)H(s)} \tag{9}$$

Let $G(s) = \frac{p(s)}{q(s)}$ and $H(s) = \frac{n(s)}{d(s)}$

where p(s), q(s),n(s) and d(s) are polynomials in s domain. Then the closed loop transfer function will be

$$W_{mi}(s) = \frac{K(s+z_1)(s+z_2)...(s+z_m)}{(s+p_1)(s+p_2)...(s+p_n)} \tag{10}$$

Since there are real and complex conjugate poles, the closed loop response of the system to a unit step input can be represented as

$$W_{mi}(s) = \frac{K\prod_{i=1}^{m}(s+z_i)}{s\prod_{j=1}^{q}(s+p_j)\prod_{k=1}^{r}(s^2+2\varsigma_k w_k s+w_k{}^2)} \tag{11}$$

Let the dominant complex poles for k=1 be $s^2+2\varsigma_1\omega_1 s+\omega_1{}^2$

Where:

ω_1 is the natural frequency and ς_1 is the damping ratio.

From (9), the roots of the characteristic equation for the dominant poles are

$$s, s_2 = -\varsigma_1\omega_1 +/- \omega_1\sqrt{\varsigma_1{}^2-1} \tag{12}$$

Thus (9) can be rewritten as

$$W_{m_i}(s) = \Psi(\omega_n, \varsigma) \tag{13}$$

Combining (8) and (10)

$$\hat{y}_{m_i}(t) = \lambda(\Phi_i, \vartheta_i, \omega_n, \varsigma) \tag{14}$$

Where:

$\lambda = [\Phi_i, \vartheta_i, \omega_n, \varsigma]$ is the input parameter vector

However, from (2) & (3) it can be seen that Φ_i, ϑ_i are dependent on the system auxiliary states Ω. Thus the developed reference output depends on the systems auxiliary states, ς and ω_n. The fuzzy decision rules consist of inputs and outputs and inputs are the command signal and significant auxiliary outputs from the system.

The outputs are the parameters in vector λ. The decision is preformed by fuzzy rule, which has the following form:

R_i: **IF** x is A_i **AND** y is B_i **THEN** z is C_i

where x and y are the inputs, z is the output and the subscript i indicates the i^{th} rule.

Corresponding to each of these modes the fuzzy parameter vector Φ_i and membership function weights ϑ_i will be changed. Subsequently, the crisp set of output vector λ is established. This process is continued for each time instant during system operation.

b) Method 2

The second method is a development inspired by the work in [9]; the desired feedback gain matrix is formulated for each mode of operation so that the eigenvalues and eigenvectors of the original closed loop system are recovered. Let us assume that there are H modes of operation. Thus for each mode of operation, the closed loop system gain matrix will change and the desired system response is identified with corresponding active poles and zeros. Based on that, fuzzy rules have been developed to meet the following from

R_i: **IF** x is A_i **AND** y is B_i **THEN** z is $C_{i1}...C_{in}$ & $D_{i1}...D_{im}$

Where:

1—n corresponds to the number of poles and

1—m corresponds to the number of active zeros.

Using a reconfigurable control gain matrix an output gain matrix K can be established such that the maximum number of closed-loop eigenvalues of the reconfigured system is the same as that of the original system.

Mathematically it is rewritten as

$$\lambda_i^m = \lambda(A_m + B_m K_m C_m) = \lambda_i \quad (15)$$

Where:

λ_i, is the eigenvalues and A, B and C are the system parameters.

The gain matrix will satisfy the following equation

$$\lambda_i^m v_i^m = (A_m + B_m K C_m) v_i^m \quad (16)$$

Thus it can be concluded that the eigenvalues and corresponding eigenvectors of the original closed loop system can be recovered regardless of the changes in the system operating modes. Instead of synthesizing the gain matrix K, the reference model can be moved such that the v_i^m is close to the original eigenvector v_i (the corresponding closed-loop eigenvector of the original system) as possible. Based on this, a new reference model structure is developed at each instant of time by changing the roots of the characteristic equation. As it can be seen from the design development, these two methods can be utilized depending on the system stage and requirement. The first method will be useful when the system can be represented in frequency domain and the second method when the system is in time domain. The time domain analysis is particularly suitable for dynamic systems with online changes. Thus it is a designer's choice to decide which of two effective methods to be utilized.

5. Plant Parameterization and Control

From the previous section the output of the plant and the reference model for a multimodal system can be represented as

$$y_i(t) = A_{m(t)} X(t) + b_{m(t)} U(t) \quad (17)$$

$$\hat{y}_{mi}(t) = \hat{W}_{mi} * S(t) \underline{\underline{\Delta}} (\Phi_i * \vartheta_i) * W_{mi} * S(t) \quad (18)$$

As the plant modal changes, the structure and dynamics of the reference model also change. Thus any adaptive certainty equivalence control law can be used to control these systems. A brief description of the proposed adaptive control technique is described next. Suppose a plant controller can be represented in terms of the parameter identification of the system. Then this approach proves that as the system identification routine reaches the actual values, then the controller will force the plant to the reference pattern such that the error asymptotically reduces to zero. For the proposed system the control law can be expressed as

$$U = \Theta^T \omega \quad (19)$$

Where:

$\Theta = \begin{bmatrix} k & \theta_0 & \theta_1^T & \theta_2^T \end{bmatrix}^T$ is the control parameter vector and $\omega = \begin{bmatrix} S(t) & y_i(t) & \omega_1^T & \omega_2^T \end{bmatrix}^T$ the regression vector.

The regression vectors are updated online based on the following equation

$$\omega_1 = \Lambda \omega_1 + LU \quad (20)$$

$$\omega_2 = \Lambda \omega_2 + L y_i(t) \quad (21)$$

where Λ is a stable matrix of order $(n-1)X(n-1)$ such that the determinant $|sI - \Lambda| = Z_m(s)$ and the vector L is defined as $L^t = [0\ldots0, 1]$.

Further, the control signal U which is structured earlier can be optimized as

$$k = -\text{sgn}(K_p) er \quad (22)$$

$$\theta = -\text{sgn}(K_p) e y_p \quad (23)$$

$$\theta_1^T = -\text{sgn}(K_p) e \omega_1^T \quad (24)$$

$$\theta_2^T = -\text{sgn}(K_p) e \omega_2^T \quad (25)$$

Thus the overall scheme can be shown as in Fig. 1.

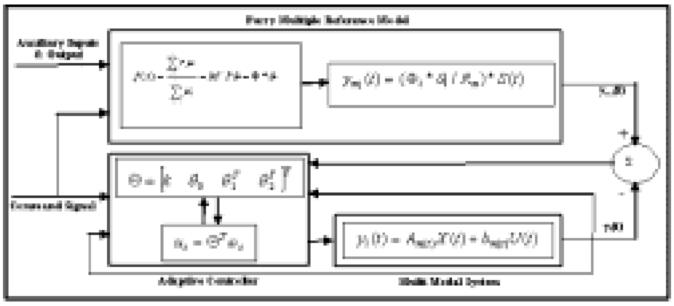

Figure 1. Proposed Controller

5.1 Stability Analysis of the FMRMAC

One of the main criteria here is to ensure that the $W_{mi}(s)$ is chosen so that the closed loop system shows asymptotic stability. This section illustrates the overall system stability of the proposed control scheme. The fuzzy parameterization of the reference model can be denoted as a parametrically changing function. From (4)-(8) and using (3) and (17),

$$\hat{y}_{mi}(t) = P_1 \vartheta_1 + P_2 \vartheta_2 + \ldots + P_{n+1} \vartheta_{n+1} \quad (26)$$

Using (26) and since P indicate parametric changes in reference model structure

$$\hat{y}_{mi}(t) = \vartheta \left[A_m y_m + b_m S(t) \right] \quad (27)$$

Where:

$$A_m = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi'_1 & \phi'_2 & \phi'_3 & \ldots & \phi'_n \end{bmatrix} \quad b_m = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ \phi'_{n+1} \end{bmatrix}.$$

It is worth noting that the coefficient matrix has been controlled by the fuzzy structure. Suppose the system output need to track the reference model structure. The error function will then be

$$e = x - x_m = \left[e \quad \dot{e} \quad \ldots \quad e^{(n-1)} \right] \in \Re \qquad (28)$$

and

$$\dot{e} = \dot{y} - \dot{y}_m = A_{m(t)} X(t) + b_{m(t)} U(t) - \vartheta(A_m y_m - b_m S(t) \qquad (29)$$

Based on (19), the control law can then be written as

$$U = \left[\theta_{12}(t) X(t) + \theta_0(t) \vartheta y(t) + k(t) \vartheta S(t)\right] \qquad (30)$$

Where:

ϑ represents the fuzzy logic membership function, and θ_{12} represents $\left[\theta_1 \quad \theta_2\right]$

If the state vector is identified every time instant through an adaptive mechanism then X(t) will be updated as a regression vector as indicated in (20) and (21). This regression vector update can then be denoted as θ_{12}. Let the desired or perfect control can be defined as (31) based on certainty control law.

$$U^* = \left[\theta_{12}{}^* X(t) + \theta_0^* \vartheta y(t) + k^* \vartheta S(t)\right] \qquad (31)$$

Where

$\theta_{12}{}^* = -A_1 / b_1$ $\theta_0{}^* = -A_m / b_1$ and $k^* = -b_m / b_1$

Theorem: Given that the fuzzy system represents the linearized reference model structure at each time instant that has the roots in the left half plane, the proposed adaptive control law provides error to move asymptotically to zero and is stable based on a Lyapunov function.

Proof: Considering (26)-(31) let the error function

$$e = X_m - X \qquad (32)$$

then

$$\dot{e} = \vartheta A_m y_m + \vartheta b_m S(t) - \left[A_1 X(t) + b_1(\theta_{12}(t) X(t) + \theta_0(t) \vartheta y + k(t) \vartheta S\right] \qquad (33)$$

Rewriting (33)

$$\dot{e} = -\vartheta A_m e + b_1 \tilde{\theta}_{12} X(t) + b_1 \tilde{\theta}_0 \vartheta y + b_1 \tilde{k} \vartheta S \qquad (34)$$

Where '~' represents the error between actual and desired values

Assuming a Lyapunov function candidate

$$V(e, \tilde{\theta}_{12} \tilde{\theta}_0, \tilde{k}) = \frac{e^2}{2} + \frac{1}{2\gamma_1}\left[\frac{\tilde{\theta}_{12}{}^2}{2}\right] + \frac{1}{2\gamma_2}\left[\frac{\tilde{\theta}_0{}^2}{2}\right] + \frac{1}{2\gamma_3}\left[\frac{\tilde{k}^2}{2}\right] \qquad (35)$$

It can be seen that if $\gamma_1, \gamma_2, \gamma_3$ are positive then (35) is positive. Now considering the derivative of the function

$$\dot{V}(e, \tilde{\theta}_{12} \tilde{\theta}_0, \tilde{k}) = e\dot{e} + \frac{1}{\gamma_1}\left[\tilde{\theta}_{12} \dot{\tilde{\theta}}_{12}\right] + \frac{1}{\gamma_2}\left[\tilde{\theta}_0 \dot{\tilde{\theta}}_0\right] + \frac{1}{\gamma_3}\left[\tilde{k} \dot{\tilde{k}}\right] \qquad (36)$$

Let

$$\dot{\tilde{\theta}}_{12} = -\gamma_1 e \operatorname{sgn} \omega(b1) \qquad (37)$$

$$\dot{\tilde{\theta}}_0 = -\gamma_2 e \vartheta y \operatorname{sgn}(b_1) \qquad (38)$$

$$\dot{\tilde{k}} = -\gamma_3 e \vartheta S \operatorname{sgn}(b_1) \qquad (39)$$

If X(t) is extracted by the regression vectors as in (20) and (21) then X(t) can be represented linearly by 'ω'. From (36) and (37)-(39),

$$\dot{V}(e, \tilde{\theta}_{12} \tilde{\theta}_0, \tilde{k}) = -Am \vartheta e \qquad (40)$$

Thus the overall system is stable in Lyapunov sense since ϑ is always positive thus making (36) negative.

5.2 Fault Tolerant Feature of the FMRMAC

Consider the case that the known faults are used to develop certain modes of operation of the plant. Further assume that the nominal conditions are the rest of the modes. If a certain mode occurs at an instant, which is a combination of faulty modes and nominal condition, then the reference model developed by the fuzzy system is a fuzzified combination of models belongs to the model set Mo. Let each mode be represented by a set of fuzzy rules. Then the model set will be the combination of the models shown in (41)

$$Mo = \sum_{i=1}^{N} \mu_i Mo_i \qquad (41)$$

The membership function weights ϑ_i in (3) represent the combination of fuzzy outputs of the mode transition which in turn affect the development of model combination at each time instant. In this sense, essence of (14) is exactly represented by the output of the fuzzy logic system. When the controller used is the direct model reference adaptive controller type the reference model structure under faulted regime will be a combination of faulty modes and the nominal ones. Based on this, from (41) and (40) it can be seen that the reference model even in case of system faults cannot be zero thus making the coefficient matrix (A_m) is always positive. This critical feature guarantees that (40) is always negative making the overall controller tolerant to faults.

6. Simulation Results and Discussions

The system to be controlled is a single flexible link manipulator for tracking the angular position. The system model is detailed in [14]. The objective is to track the angular position. Command signal is applied for eight seconds and the tip load is varied arbitrarily at different time instant. The observation of the tracking error and the output (angular position path of the single link manipulator) is shown in two cases. These cases are simulated based on two different methods that are proposed in section four.

(a) Case 1

In this case, the tip load of this manipulator is changed at different time instants as in Table 1. The reference model representation as generalized in method one of section four has the following specific form,

$$W_m(s) = \frac{\omega_n^{\ 2}}{s^2 + 2\varsigma\omega_n + \omega_n^{\ 2}} \tag{42}$$

where the value of ς is set to 0.7. The value of ω_n will be evaluated at every instant of time depending on the two auxiliary inputs, the system parameter a_1 and the load torque.

The system parameter a_1 has been derived from one of the operating state that is induced from output measurements. The importance of this state is that it is closely related to the control value. Thus the only requirement to develop a fuzzy structure is to generate auxiliary states and the output. In order to develop the fuzzy multiple reference model generator the first step is to determine the range of the inputs and the outputs. By simulating and studying the process it was found that for the range of load torque [0, 24], the closed loop system response was best by keeping ω_n between [0, 24].

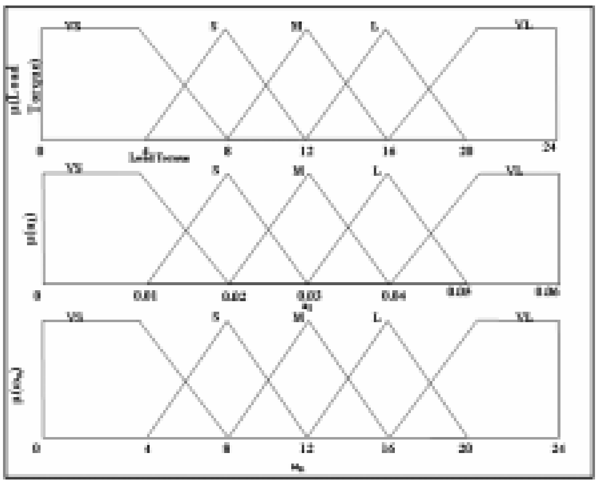

Figure 2. Membership function details for inputs and Outputs.

The ranges of fuzzy membership functions are as shown in Fig. 2. The rule base was created depending on system operation modes, which is divided into five operating regions; Very Small(VS), Small(S), Medium(M), Large (L) and Very Large(VL). The rule base created for this specific example is as in Table 2. Thus as an example, a load torque of 10 Nm has membership functions of 0.5 in S and 0.5 in M (μ_S=0.5 and μ_M=0.5). Further, an estimated value of a_1 of 0.05 has membership functions of 0 in L and 1 in VL; accordingly the following rules will be fired.

Table 1. Tip Load Variation (Case 1)

Time Range (sec)	0-3	3-3.5	3.5-6	6-6.8	6.8-8.0
Load Torque (Nm)	0	15	0	20	0

Table 2. Rule Base (Case 1)

ω_n	**VS**	**S**	**M**	**L**	**VL**
VS	S	S	M	L	VL
S	S	S	M	L	VL
M	S	S	M	L	VL
L	S	S	M	L	VL
VL	S	S	M	L	VL

Rule # 9 (S/L) with μ_1=0 in S (ω_1=5)
Rule # 10 (S/VL) with μ_2=0.5 in S (ω_2=5)
Rule # 14 (M/L) with μ_3=0 in S (ω_3=6)
Rule # 15 (M/VL) with μ_4=0.5 in M (ω_4=6)

Where S/L means when load torque is small and a_1 is large then the membership function is 0 and the value of ω_n is 5. Applying the defuzzification rule the extraction process of ω_n is as follows

$[\mu_1(\omega_1) + \mu_2(\omega_2) + \mu_3(\omega_3) + \mu_4(\omega_4)]/(\mu_1 + \mu_2 + \mu_3 + \mu_4)$

which makes

$$\omega_n = \frac{0(5) + 0.5(5) + 0(6) + 0.5(6)}{1} = 5.5 \tag{43}$$

Based on this ω_n, the second order reference model can be generated as

$$W_m(s) = \frac{30.25}{s^2 + 11\omega_n + 30.25} \tag{44}$$

Considering the proposed approach using method one, for the following manipulator tip load, the fuzzy mapping is used for the multiple reference models. Fig. 3 shows the position trajectory plot comparing a single reference model in which ω_n is kept as 5 and the above-mentioned approach during the load variation (mode change) which various modes of the system for duration of eight seconds. It can be seen that the single reference model approach shows instability while the proposed approach shows successful position trajectory tracking as illustrated in Fig. 3. This figure also shows the trajectory error and the model error. The trajectory error is the error between the input command and the output of the system (which is the manipulator tip position) and the model error is the error between the outputs of the reference model and the system.

(b) Case 2

In this case, the tip load of this manipulator is changed at different time instants as in Table 3.The reference model structure based on method two is as in (45).

Table 3. Tip Load Variation (Case 2)

Time Range (sec)	0-3	3-3.5	3.5-6	6-6.8	6.8-8.0
Load Torque (Nm)	10	10	0	7	0

$$W_m(s) = \frac{(\omega_n{}^2 / a)(s + a)}{s^2 + 2\varsigma\omega_n + \omega_n{}^2} \qquad (45)$$

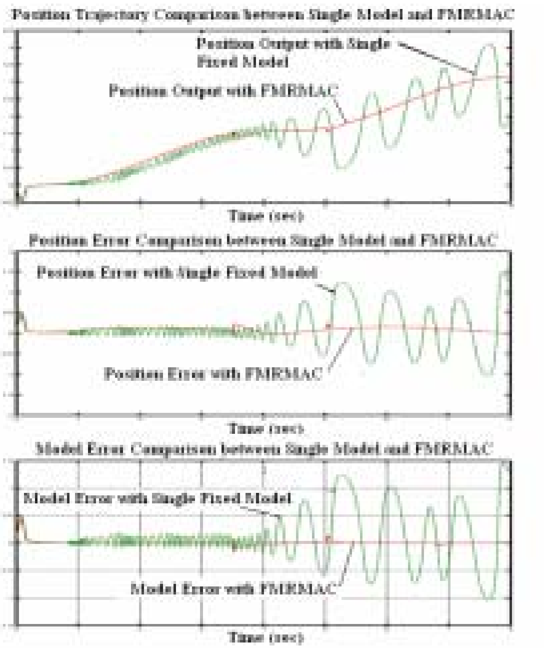

Figure 3. Position tracking comparison (Case 1): a) output, b) trajectory error and c) model error.

Table 4. Table Rule Base (Case 2)

Load Torque	ω_n \ a_1	VS	S	M	L	VL
	VS	S	S	M	L	S
	S	S	S	M	L	S
	M	L	S	M	L	S
	L	S	S	M	L	VL
	VL	S	S	M	L	VL

Where a is a factor that can be determined based on the system dynamics requirement from system auxiliary state. The details regarding the extraction process of reference model using (28) is as follows. At first the system gain matrix has been generated based on the static closed loop control. Then the desired eigen values and vectors are extracted for various system modes of operation. The basis of the fuzzy reference model generator is to move the reference model structure such that the desired eigen vector is close to the system eigen vectors as much as possible. To this end, the dominant poles and zeros are changed in this case unlike as in Case 1. Changing the value of constant a in (45) depends on the natural frequency ω_n. This leads to a change in the zero location. As it can be assessed, the value of the constant *a* has been synthesized from the gain matrix required to move the reference model structure. The location of the poles is also changed by a change in the value of ω_n. The effect of the change in the natural frequency and the constant *a* derives new reference model structure at every time instant. The set of fuzzy knowledge base is shown in Table 4. It is observed that the transient response of the system with one zero and two poles will be affected by locations of zeros.

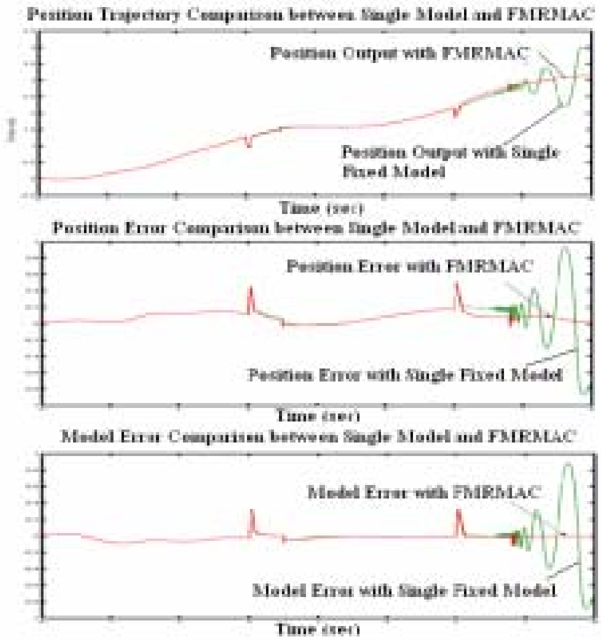

Figure 5. Position tracking comparison (Case 2), a) output, b) trajectory error and c) model error.

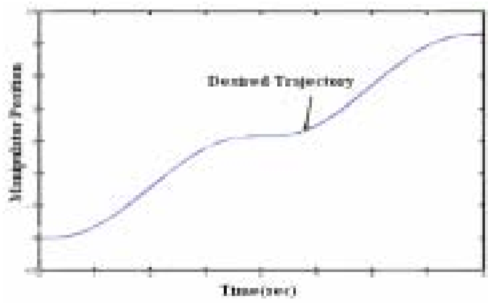

Figure 6. Position tracking of the single reference model.

The ω_n value of the fixed model is kept constant at 2 for this case. For fuzzy reference model, ω_n is varied depending on the fuzzy rules. It is worth noting that even though fuzzy model generation requires various known ω_n values, the model representation of each of them separately gives the same results as when ω_n is equal to

2. This is due to the fact that each of these models with various individual ω_n values fails to perform at one point or the other in the trajectory. Thus the simulation results for each models with various ω_n values are not shown in both these cases. Fig. 4 shows the responses of the system position output with fuzzy reference model generation and a fixed model. It can be seen that the fixed model shows the instability with the error increasing rapidly as in Case 1, while the response of the proposed method achieved its objective. For all these cases Fig. 5 shows the desired trajectory.

From the above results and many other similar results it can be concluded that the FMRMAC performs well when there is a change in the plant mode of operation. However, if the plant operates in only one mode then the single reference model adaptive controller performs very well. In such a condition, it is worth noting that the fuzzy reference model adaptive controller, even though it may perform equally well as a single reference model adaptive controller, generates an additional computational burden for the closed loop control.

7. Conclusions

A novel FMRMAC scheme for multimodal and 'Jump' Systems has been developed. The feasibility and effectiveness of the proposed scheme have been investigated by applying it to an important and challenging practical system; a position control of a single link flexible robotic manipulator. Investigation results showed that the proposed FMRMAC scheme outperformed both traditional and single reference model adaptive controllers. The scheme provides soft switched fuzzy reference model and was found stable, especially at the modal boundaries when the 'hard switching' mathematical approach fails. Further the scheme is computationally feasible, and fault tolerant. Unlike other approaches to multiple model adaptive control, the proposed approach can be used with a direct model adaptive control without an explicit identifier.

8. References

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