Experiment #2: Telescopes And Microscopes

Purpose: To measure the focal lengths of converging and diverging lenses and use this information to construct an astronomical telescope, a Galilean telescope, and a microscope.

Equipment:

- Optical bench and pin accessory kit
- Two converging and one diverging lens (see note below)
- Light source
- Transparent ruler
- Erasable fine-tipped transparency felt pen (which can draw on the lenses)

NOTE ABOUT LENSES—One converging lenses should have focal length in the 5-10 cm range and the other in the 10-15 cm range. A quick way of estimating this is to image a ceiling light onto a table and measure the distance between the lens and table. The diverging lens should have a focal length intermediate between the two converging lenses; in a darkened room you would then be able to form the image of a distant light with the combination of the diverging lens and the stronger converging lens. If you are unable to do this, i.e., if the diverging lens is stronger than both converging lenses, then you will need to mount all 3 lenses together in Step 3 of Part 1 and instead use the equation

$$1/f_1 + 1/f_2 + 1/f_3 = 1/F$$

in Step 4 of Part 1. On the other hand, if the diverging lens is weaker than both converging lenses, combine two diverging lenses together.

Part 1—Focal Lengths of the Lenses

Discussion:

A converging lens is thicker at its center than at the periphery. Parallel incident rays pass through the lens and converge to a real focus on the opposite side. A diverging lens is thinner at the center than at the periphery. Parallel incident rays pass through the lens and diverge on the opposite side. These rays appear to originate from a virtual focus on the same side of the lens as the incident rays.

The principal axis of a lens is a line drawn through its center perpendicular to the plane of the lens. The principal focal point F of a converging lens is that point on the principal axis where all incident rays parallel to the principal axis intersect after passing through the lens; for a diverging lens the principal focal point F is the point on the principal axis from where the diverging rays appear to originate from. There is also a secondary focal point F' which in both cases is located as far from the lens as the principal focal point is, but on the opposite side of the lens. The focal plane of a lens is the plane perpendicular to the principal axis which contains the principal focal point.

The focal length f of a lens is the distance along the principal axis from the center of the lens to its principal focal point. The power of a lens, measured in diopters, is the reciprocal of the focal length in meters, i.e., $D \equiv 1/f$.

The location and size of an image may be determined by tracing rays from points on the object to corresponding points on the image. It is enough to use two rays from any point on the object to locate the position of its image. The procedure is slightly different depending on whether you are using a converging or a diverging lens—see Fig. 1(a)–(b). The object distance o is defined to be the distance from the object to the center of the lens, while the image distance i is the distance from the center of the lens to the image.

From any point on the object, ray 1 is drawn parallel to the principal axis. For a converging lens, the ray is refracted by the lens and passes through the principal focal point F on the other side of the lens. For a diverging lens, the ray diverges after emerging from the lens, as if it were coming from the principal focal point F on the incident side of the lens. Ray 2 from the object point is drawn so that it passes through the center of the lens undeflected in both cases.

If the two rays intersect after they emerge from the lens, the point of intersection is a real image of the object point, which means that the image may be seen on a screen placed at that point. If they are diverging after they emerge from the lens, then imagine projecting the rays backward through the lens until they intersect. This point, from which the rays seem to be emerging, is the position of the virtual image. The virtual image can be seen through the lens by an observer but cannot be captured on a screen.

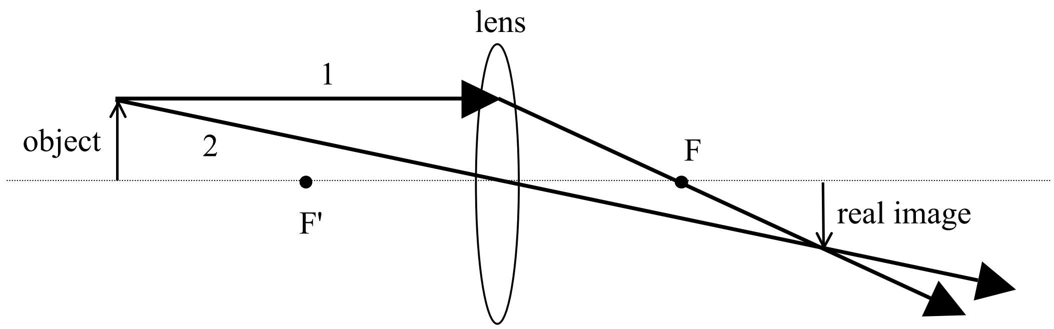

Fig. 1(a). Converging lens with $o > f$

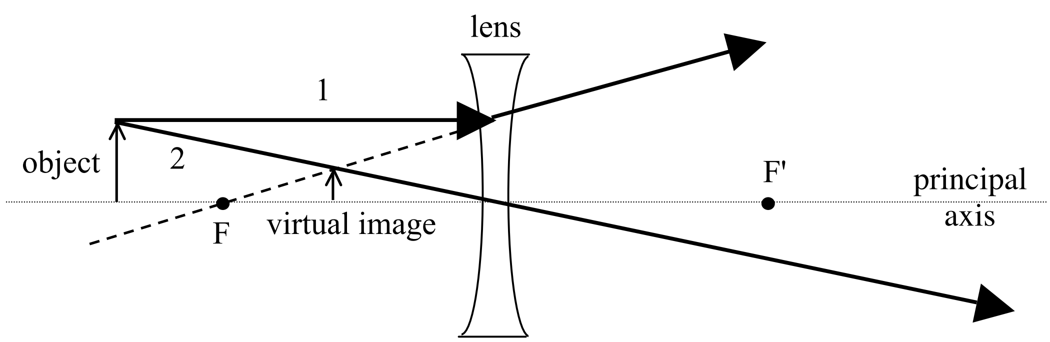

Fig. 1(b). Diverging lens with $o > |f|$

The size and position of an image may be determined by a ray tracing diagram as above or, as you will see in the lab procedure, algebraically. In doing this, we will follow several sign conventions for the different distances in the problem:

1. The focal length f is positive for a converging lens and negative for a diverging lens.
2. The object distance o is positive for a real object, that is if the object is on the same side of the lens as where the light is coming from; o is negative for a virtual object when the object appears to be on the opposite side of the lens as the incident rays (e.g., the image formed by the objective of a Galilean telescope is a virtual object for the eyepiece).
3. The image distance i is positive for a real image, which occurs when the image is on the opposite side of the lens as where the light is coming from. If the image and incident rays are on the same side, one has a virtual image and i is taken to be negative.

<u>Procedure</u>:

We will determine the focal length of two converging (convex) lenses by direct measurement. We will then measure the focal length of a converging-diverging lens combination and from this determine the focal length of the diverging (concave) lens.

1. Mount the light source, object (cutout arrows), one of the converging lenses, and an image screen (a piece of carboard works well) on the optical rail in that order. Start with the distance from the object to the lens being about 20 cm, then vary the distance from the lens to the screen (by moving the screen only) until a sharp image is seen. Record the object distance o and image distance i, and then use the thin lens equation to calculate the focal length f:

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}.$$

Repeat two more times with o approximately 5–10 cm larger or smaller than before (but never less than f). You should get roughly the same answer for the focal length in all three cases. Average these three values to get one final value for the lens.

2. Now repeat step 1 for the other converging lens. MAKE SURE TO KEEP TRACK OF WHICH LENS IS WHICH! The lenses should have labels on them—write down those codes next to the focal lengths in your notes.

3. Mount the stronger of the two converging lenses (i.e., the one with the shortest focal length) and the diverging lens together into one lens holder. A strip of tape placed around the periphery may make this task easier. Be careful not to drop the lenses. Repeat the procedure of step 1 for this combination to get its combined focal length F.

4. Now use the known focal length f_1 of the strongest converging lens from steps 1–2 and of the combination F from step 3 to determine the unknown focal length f_2 of the diverging lens according to the formula

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F}.$$

Should your answer be positive or negative? Give a reason why in your report.

Part 2—Construction of Telescopes and Microscopes

Discussion:

We will first discuss the most basic optical instrument, the “magnifying glass,” also called a “simple magnifier.” The normal human eye can focus a sharp image of an object on the retina if the object is located anywhere from infinity (such as stars) to a certain point called the near point N. The standard distance *n* to the near point is taken in textbooks to be 25 cm. However, for most young people like yourselves, *n* is shorter than that, as we will determine by direct measurement. The retinal image of an object closer to the eye than the near point is fuzzy. If an object of height h_o is placed at the near point then the angle (in radians) subtended by it at the eye is

$$\tan\theta = \frac{h_o}{n}$$

as can be seen from Fig. 2.

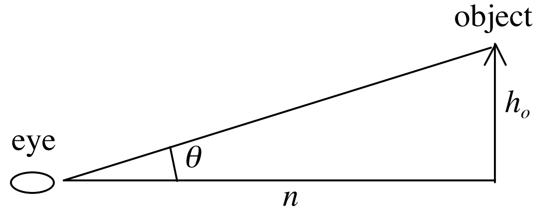

Fig. 2. Maximum magnification of the naked eye.

Assuming that $h_o << n$, then the angle is small enough that $\tan\theta \approx \sin\theta \approx \theta$, so that

$$\theta \cong \frac{h_o}{n}.$$

If the object is now viewed through a convex lens, in such a way that the object distance *o* is slightly less than the focal length *f*, a magnified, virtual, erect image is observed. If the height of the image is h_i, then the angle it will subtend at the eye is

$$\theta' \cong \frac{h_i}{i}$$

as can be seen from Fig. 3, which Halliday, Resnick, and Krane got wrong on page 938!

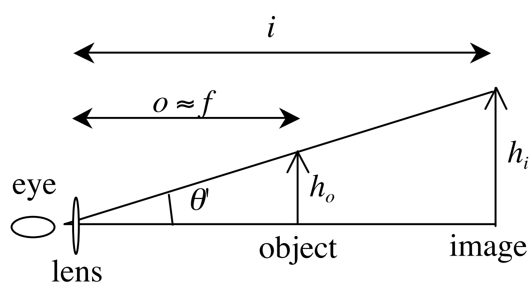

Fig. 3. Magnification of a simple magnifier

But from the similar triangles in this figure, we see that

$$\frac{h_i}{h_o} = \frac{i}{o} \cong \frac{i}{f} \quad \text{so that} \quad \theta' \cong \frac{h_o}{f}.$$

The angular magnification is defined as the increase in angular size,

$$m \equiv \frac{\theta'}{\theta} = \frac{n}{f}.$$

Since a realistic lower limit on the focal length is a few centimeters in order to avoid getting aberrations, simple magnifiers typically have magnifications no greater than 10.

The simple telescopes and microscope which you will build here are basically two-lens optical systems, in which a converging lens (the objective) is used to form a real image of the object, whether it be a tiny bug or a distant planet. A second lens called the eyepiece, which can be either converging or diverging, is then used to examine the image formed by the objective.

In an astronomical (or Newtonian) telescope, the objective ("obj") forms a real inverted image I of a distant object O (such that $o \approx \infty$) in its focal plane, as shown in Fig. 4 below. The angular size of the image formed by the objective is given by

$$\theta_{obj} \cong \frac{h_i}{f_{obj}}$$

where h_i is the height of the image and f_{obj} is the focal length of the objective. This is also the angular size of the distant object, as we see in the following diagram.

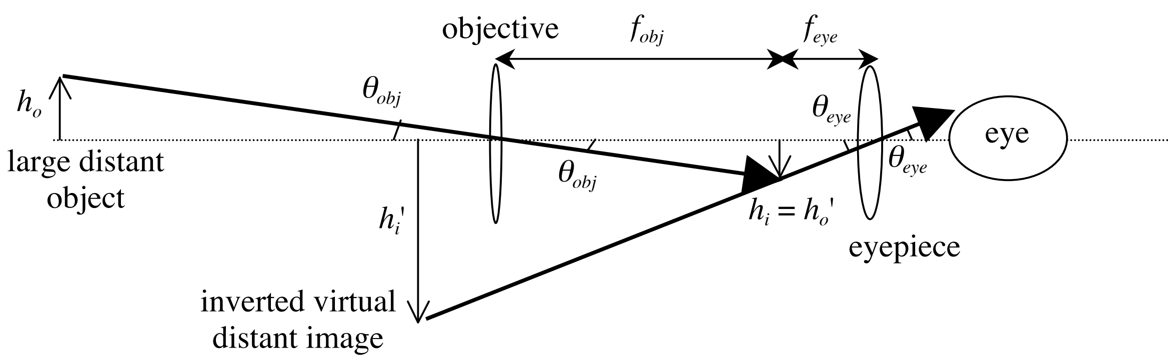

Fig. 4. Astronomical Telescope

The image I of the objective is the real object O' of height $h_o' = h_i$ for the eyepiece, which is used as a simple magnifier. The eyepiece then produces (as in Fig. 3) a virtual image I' which is magnified in size to h_i' but remains inverted. The angular size of the image seen through the eyepiece ("eye") is seen from Fig. 4 to be

$$\theta_{eye} \cong \frac{h_i}{f_{eye}}$$

where f_{eye} is the focal length of the eyepiece. The image I formed by the objective is very near the focal point of the eyepiece, so that the distance L between the two lenses is approximately equal to the sum of their focal lengths, $L = f_{obj} + f_{eye}$. The total angular magnification of the telescope is

$$M \equiv \frac{\theta_{eye}}{\theta_{obj}} = \frac{f_{obj}}{f_{eye}}$$

and evidently the objective must be weaker (i.e., have a longer focal length) than the eyepiece in order to give magnification ($M > 1$).

The astronomical telescope produces an inverted image, which is okay for looking at stars but inconvenient for use on land. So for the latter purpose, a terrestial (or Galilean) telescope can be employed instead, resulting in an erect image. A terrestial telescope is constructed using a diverging lens for the eyepiece rather than a converging lens, as shown in Fig. 5 below.

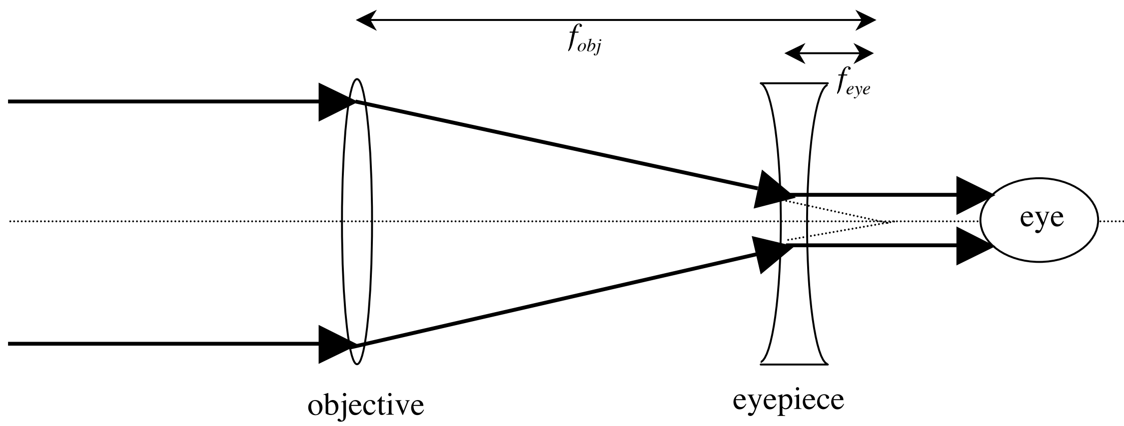

Fig. 5. Terrestial telescope

This figure assumes that you are looking at an object which is very far away (i.e., near infinity so that the incoming rays are very nearly parallel). The resulting image is also at infinity, which corresponds to the relaxed eye position. Note that $f_{obj} > 0$ for the converging objective lens, while $f_{eye} < 0$ for the diverging eyepiece. The total angular magnification of the telescope is, just as for the astronomical telescope,

$$M = \left| \frac{f_{obj}}{f_{eye}} \right|$$

so that we again require the objective to be weaker (i.e., have a longer focal length) than the eyepiece in order to give magnification. Also, the distance L between the two lenses is again approximately equal to the sum of their focal lengths, but since $f_{eye} < 0$, this means that the lens separation equals the difference in the absolute values of their focal lengths, $L = |f_{obj}| - |f_{eye}|$. Thus, an added advantage of the terrestial telescope is that it is more compact than the astronomical telescope.

Finally, in a microscope a small object is placed near, but just beyond, the focal point of the objective so that an enlarged, real, inverted image I is formed, which becomes a real object O' for the eyepiece. The eyepiece then acts as a simple magnifier, as in the astronomical telescope, to produce a final image I' which remains inverted—cf. Fig. 6 below. If s is the distance between the focal points F_{obj} and F_{eye}', while h_o and h_i are the heights of the object and image produced by the objective, then the lateral magnification of the objective is

$$m_{obj} \equiv \frac{h_i}{h_o} = \frac{s}{f_{obj}}$$

based on the similar triangles in Fig. 6. The total magnification of the microscope is the product of the lateral magnification m_{obj} of the objective and the angular magnification $m_{eye} = n/f_{eye}$ of the eyepiece (as in Fig. 3),

$$M \equiv m_{obj} m_{eye} = \frac{s}{f_{obj}} \frac{n}{f_{eye}} = \frac{(L - f_{obj} - f_{eye})n}{f_{obj} f_{eye}}$$

where, as for the telescopes, L is the distance between the lenses.

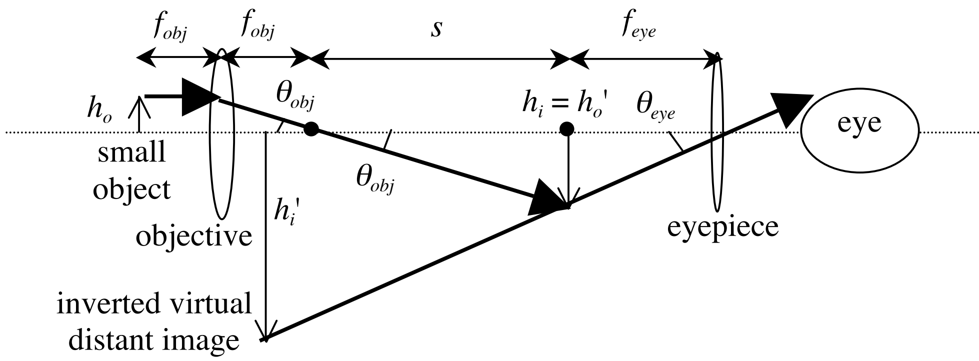

Fig. 6. Microscope

<u>Procedure</u>:

1. Build an astronomical telescope using the two converging lenses from part 1 of this lab separated by a distance equal to the sum of their focal lengths. Use the stronger lens as the eyepiece. Bring your eye up close to the eyepiece and look through it at a distant object; it should appear inverted. If you normally wear glasses, keep your glasses on. Slightly adjust the distance between the two lenses to focus the telescope. (Note: If your eye is too far from the eyepiece, you can get fooled into thinking the various instruments constructed in this lab are focused when they actually are not.)

2. Determine the experimental magnification of your telescope by looking at two parallel lines which you have drawn on the blackboard across the room. Hold up a ruler at about arm's length (but not so far that you cannot read the scale!), look through the telescope with one eye, and measure the apparent spacing between the two lines with your other eye. (If you cross your eyes you may be able to overlap the two images, making this task easier.) Now move your head slightly so that you are looking at the two lines with your naked eye, and again measure the apparent spacing between the two lines <u>without</u> changing the distance from the ruler to your eye. The ratio of the spacings measured with and without the telescope is the experimental magnification. Compare the result to the theoretical magnification which equals the absolute value of the ratio of the two focal lengths of the lenses. (Accuracies are not really good enough to warrant a calculation of percent error. Instead discuss in your report whether the agreement seems reasonable and why the accuracy isn't expected to be terribly high.) What happens if you turn the telescope around and look through the wrong end of it? Explain why this happened in your report.

3. Next, build a terrestial telescope by using the weaker of your two converging lenses as the objective and the diverging lens as the eyepiece, separating the two lenses by the difference in the absolute values of their focal lengths. Again bring your eye up close to the eyepiece and look through it at a distant object, and slightly adjust the distance between the two lenses to focus the telescope. This time things should appear upright.

4. Repeat step 2 for your terrestial telescope.

5. Finally, build a microscope according to the following procedure using the two converging lenses but now the objective should be the strongest lens, unlike for the two telescopes. First mount the ruled translucent scale a few centimeters beyond the (secondary) focal point of the objective lens, to serve as an object to examine; measure the distance *o* between this object and the objective. Then use the thin lens equation

to compute the distance *i* from the objective to the image produced by it. Now position the eyepiece at a distance from the objective equal to $i + f_{eye}$. Finally, bring your eye up close to the eyepiece (with your glasses on) and slightly adjust the position of the ruled scale until you see a focused image of it. (You and your partner may have to each do this separately, since your eyes may be slightly different.) Note that the microscope will invert the image relative to the object.

6. Determine the experimental magnification of your microscope as follows. Look at an adjacent pair of rulings on the object screen through the microscope, back your eye up slightly, and use a felt pen to mark the apparent positions of the two rulings directly on the eyepiece lens. Now measure the separation between your two marks with a ruler. Also measure the actual separation between the two rulings on the screen. The ratio of these two separations is the magnification.

7. To get the theoretical magnification of the microscope, first measure your near point distance. To do so, hold up a plastic ruler to the bridge of your nose with the zero end at your nose. Gradually bring a small object which has writing on it along the ruler toward your eye (with your glasses on). Find the shortest distance at which you can still comfortably focus on the writing (i.e., without having to strain so much that you get a headache: be reasonable!). Use your near point distance *n* to compute the theoretical magnification as $M = (L - f_o - f_e) \times n / (f_o \times f_e)$ where *L* is the measured distance between the two lenses. Compare to the experimental magnification found in step 6.

<u>Supplementary Problem</u>:

A converging lens and a diverging lens both have a focal length whose absolute value is 10 cm and they are separated from each other by 10 cm. An object is placed 15 cm away from the diverging lens (i.e., 25 cm away from the converging lens). Where is the image formed by the converging lens located? Is it real or virtual? Is it erect or inverted? How many times bigger or smaller is it than the object (i.e., what is the magnification)?