Experiment #10: LR & RC Circuits Frequency Response

EQUIPMENT NEEDED

- *Science Workshop Interface*
- Power Amplifier
- (2) Voltage Sensor
- graph paper (optional)
- (3) Patch Cords
- Decade resistor, capacitor, and inductor

PURPOSE

The purpose of this laboratory activity is to study the response of *RC* and *LR* circuits to an alternating voltage at different frequencies by examining the current through the circuit as a function of the frequency of the applied voltage. The concepts of phase shifts and low-pass and high-pass filters will also be studied.

THEORY

We will use the following symbols:

ε: Voltage output of Signal Generator (also called the Source voltage or Applied Voltage)

V_R, V_C, V_L: Voltage across Resistor, Capacitor, Inductor

i: Current through each component

BASIC PRINCIPLES

We will deal with circuits with a single loop. So the current is always the same through all components in the circuit. If we can find the current through one component, the same current is flowing through the other components also.

1. V_R and i are always in phase with each other. They are related by Ohm's law. The amplitudes V_{Rm} of the voltage and i_m of the current are also similarly related:

$$V_{Rm} = R\, i_m \tag{1}$$

2. i lags ε in phase. The amplitudes are related by:

$$V_{Lm} = \omega L\, i_m = X_L\, i_m \tag{2}$$

3. i leads ε in phase. The amplitudes are related by:

$$V_{cm} = \frac{1}{\omega C} i_m = X_C \, i_m \tag{3}$$

The amplitude of the AC current (i_m) in a series *LR* or *RC* circuit is dependent on the amplitude of the source voltage (ε_m) and the impedance (Z).

$$i_m = \frac{\varepsilon_m}{Z} \tag{4}$$

Since the impedance depends on frequency, the current varies with frequency:

For a series RC circuit, the impedance is given by

$$Z = \sqrt{X_C^{\,2} + R^2} \tag{5}$$

and for a series LR circuit, it is given by

$$Z = \sqrt{X_L^{\,2} + R^2} \tag{6}$$

where

$$X_C = \text{capacitive reactance} = \frac{1}{\omega C} \tag{7}$$

$$X_L = \text{inductive reactance} = \omega L \tag{8}$$

R = resistance, and

ω = angular frequency = $2\pi\nu$ (ν = frequency).

The two circuits are shown below:

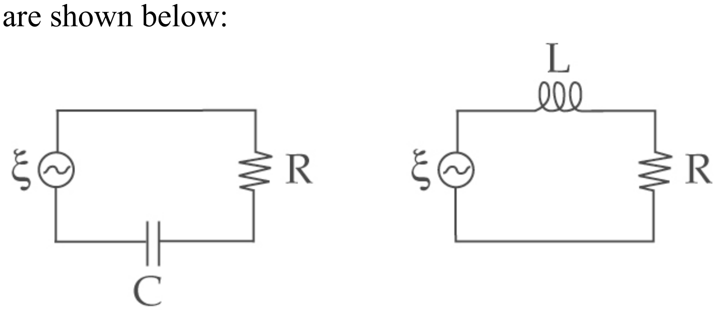

Figure 1

The output of the voltage source (such as a signal generator) is an alternating voltage:

$$\mathcal{E} = \mathcal{E}_m \sin(\omega t) \tag{9}$$

$\mathcal{E}m$ is the amplitude.

The current in the circuit is also alternating at the same frequency, but is phase shifted with respect to the source voltage

$$i = im \sin(\omega t - \phi) \tag{10}$$

<u>Note: The negative sign in equ. (8) does not mean that the phase shift is always negative. The phase shift (-ϕ) can be negative or positive depending on the circuit.</u>

<u>Also remember that the circuit has a single loop, so the current at any instant is the same all the components.</u>

The phase shift between the source voltage and the current is given in terms of the circuit components and the frequency:

RC circuit: $$\tan\phi = -\frac{1}{\varpi RC} \tag{11}$$

LR circuit: $$\tan\phi = \frac{\varpi L}{R} \tag{12}$$

Note: Since there is a negative sign before ϕ in equ. (8), these two expressions mean that the phase shift is positive for the RC circuit, and negative for the LR circuit, meaning that the current leads the voltage in the RC circuit, and the current lags the voltage in the LR circuit.

Qualitative Features: The statements below will be understood only if you have all the equations (1 – 10) in front of you, on a separate sheet, while you are reading. As you read them, you should be able to spot which equation specifically is responsible for each statement, and why.

RC Circuit:

1. At low frequencies, the capacitive reactance dominates over the resistance. So the signal voltage is dropped off mostly across the capacitance. It is as though the capacitor is offering more effective resistance than the resistor. In the extreme case, at zero frequency, the reactance is infinite, the current is zero, and all the voltage is across the capacitor.

2. At high frequencies, the capacitive reactance becomes negligible; most of the signal voltage is across the resistor. At infinite frequency, the capacitive reactance is zero, and it is as though there is no capacitor in the circuit, or the capacitor is shorted out.

3. The phase angle also behaves similarly. At low frequencies, the phase shift becomes closer to $\pi/2$, as it should for a pure capacitance with no resistance in the circuit. At high frequencies, the phase shift approaches zero, and the circuit behaves like a purely resistive circuit.

4. These statements are generally summed up by saying that the capacitor acts like a block for low frequencies, but like a short for high frequencies.

RL Circuit:

5. At high frequencies, the inductive reactance dominates over the resistance. So the signal voltage is dropped off mostly across the inductance. It is as though the inductor is offering more effective resistance than the resistor. In the extreme case, at infinite frequency, the reactance is infinite, the current is zero, all the voltage is across the inductor.

6. At low frequencies, the inductive reactance becomes negligible, most of the signal voltage is across the resistor. At zero frequency, the inductive reactance is zero, and it is as though there is no inductor in the circuit, or the inductor is shorted out.

7. The phase angle also behaves similarly. At high frequencies, the phase shift becomes closer to $\pi/2$, as it should for a pure capacitance with no resistance in the circuit. At high frequencies, the low shift approaches zero, and the circuit behaves like a purely resistive circuit.

8. These statements are generally summed up by saying that the inductor acts like a block for high frequencies, but like a short for low frequencies.

PROCEDURE

In this activity the Power Amplifier (Signal Generator) produces an alternating current through the circuit. The amplitude of the current depends on the impedance in the circuit, which varies with frequency. The Signal Generator controls the frequency. The current can be determined from the ratio of the resistor voltage to the resistance. A Voltage Sensor measures the voltage drop (potential difference) across the resistor in the circuit. The other Voltage Sensor measures the voltage drop across the capacitor (*RC* circuit) or the inductor (*LR* circuit).

So at each frequency, you will measure the following quantities:

1. Amplitude $\mathcal{E}_m$ of the source voltage

2. Amplitude V_{Rm} of the voltage across the resistor

3. Amplitude V_{Lm} of the potential across the inductor (LR circuit) of V_{Cm} of the potential across the capacitor (RC circuit)

You will use the Signal Generator to change the frequency of the applied voltage. You will investigate the phase relationship between the applied voltage and the resistor voltage as you vary the frequency. You will also determine the amplitude of the current through the resistor and the plot current vs. frequency. The *Science Workshop* program collects and displays the applied voltage, and the resistor voltage, and the voltage across the inductor or capacitor. Remember that the voltage across the resistor and the current through the resistor are always in phase. You will compare the theoretical results to your measured results. The procedure will be, at each frequency:

1. Use the known value of R and the measured value of V_{Rm} and equ. (1) to find the amplitude i_m of the current.

2. Use the measured values of V_{Lm} (*RL* circuit) and V_{Cm} (*RC* circuit) and the value of i_m found in step 1 above to calculate X_L (RL circuit) and X_C (RC circuit). These will be your measured values of the inductive and capacitive reactances of the circuit.

3. Now use equs. (7) and (8) to verify these values.

4. Use equ. (4) to find the experimental value of the impedance Z, and then verify these against the theoretical values given by equ (5) and equ (6).

5. Measure the phase shift between $\mathcal{E}$ and V_R. From these measurements, verify equs (11) and (12), and also statements (2) and (3) in the BASIC PRINCIPLES.

Now plot V_{Rm}, and V_{Lm} or V_{Cm} as functions of the frequency. Verify the qualitative statements (1 – 8) which summarize the high-pass and low-pass filter characteristics of these circuits.

PART IA: Computer Setup

- Connect the *Science Workshop* interface to the computer, turn on the interface, and turn on the computer.

- Connect the circuit according to Fig. 1 (You have to do both circuits one after the other). For the LR circuit, start with L = 500 mH, and R = 100 Ω. For the RC circuit, start with R = 10 kΩ and C = 0.5 μF.

- Connect the Power Amplifier to Analog Channel C. Plug the power cord into the back of the Power Amplifier and connect the power cord to an appropriate electrical outlet.

- Connect one Voltage Sensor to Analog Channel A and one to Analog Channel B. The banana plugs at the end of the Voltage Sensor A should be connected across the resistor. Voltage Sensor B should similarly be connected across the inductor of capacitor. The voltage measured at Analog Channel A is related to the current through the resistor by $I = \frac{V_R}{R}$. The amplitudes of the voltage measured in Analog Channel B will related to the amplitude of the current according to equ (2) for the RL circuit, or (3) for the RC circuit.

- Open the *Science Workshop* document titled as shown:

  RC2.sws

- The document opens with a Scope display of Voltage and the Signal Generator window which controls the Power Amplifier. The Scope display is set to show the applied (output) voltage (Channel C), the resistor voltage (Channel A), and the inductor or capacitor voltage (Channel B)

- Note: For quick reference, see the Experiment Notes window. To bring a display to the top, click on its window or select the name of the display from the list at the end of the Display menu. Change the Experiment Setup window by clicking on the **Zoom** box or the **Restore** button in the upper right hand corner of that window.

- The **Sampling Options…**for this experiment are: **Periodic Samples** = **Fast** at 2500 Hz (set by the Sweep Speed control in the Scope display).

- The Signal Generator is set to output 2.00 V, sine AC waveform, at 10.00 Hz. In the experiment, you will vary the frequency over the range 10 Hz – 200 Hz.

- The Signal Generator is set to **Auto** so it will start automatically when you click **MON** or **REC** and stop automatically when you click **STOP** or **PAUSE**.

- Arrange the Scope Display and the Signal Generator window so you can see both of them. Enlarge the scope display as much as possible, keeping the signal generator and the monitor window visible.

PART II: Sensor Calibration and Equipment Setup

- You do not need to calibrate the Power Amplifier of Voltage Sensor.

PART III: Data Recording

1. Turn on the power switch on the back of the Power Amplifier.

2. Click on the **MON** button to begin monitoring data. The Signal Generator will start automatically. Identify the traces of the three signals that you see in the Scope. The color scheme should tell you which signal belongs to which channel.

3. In the Scope display, click the **Smart Cursor** button. The cursor changes to a cross-hair. Move the cursor/cross-hair to a peak of the voltage V_R across the resistor. Record the voltage that is displayed next to the Input Menu button for Channel A. Record the voltage in the Data Table. This is the value V_{Rm}. Similarly record $\mathcal{E}_m$ from the signal for Channel C and V_{Lm} or V_{Cm} from Channel B. This will help you find the current i_m, the Reactances, and the impedance at this frequency.

4. **To find the phase shifts:** You will need to find the phase shift between V_R and ε in both circuits. This will help you to verify the BASIC PRINICPLES statements 1 and 2.

    a. The horizontal axis on the scope measures time, and a phase shift is proportional to a time shift. So the time between two successive peaks of a signal corresponds to a total phase shift of 2 π radians.

    b. Measure the time, using the smart cursor, between successive maxima of the source output. Note: Verify that this time corresponds to the inverse of the frequency, since that is already set on the signal generator. Similarly measure the time between successive maxima of V_R. This should give you the same value.

    c. Now, note that, for the LR circuit, the maximum in V_L occurs before the closet maximum in V_R occurs. Remember that V_R and i are in phase. Therefore, we conclude that the voltage across the inductor leads to current through the inductor.

    d. Now measure the time period between $\mathcal{E}$ and V_R, proportionally find the phase shift between current and source voltage, and verify equations (11) and (12).

5. Now change the frequency. In the Signal Generator window, click on the **Up** arrow to increase the frequency by 10 Hz. Fill the new frequency (20 Hz) in the Data Table. Repeat steps 1 – 4.

6. Repeat the process for at least 10 frequencies between 10 Hz and 200 Hz. In each case, you can adjust the sweep speed (horizontal scale) and the vertical resolution (Volts/div) on the scope, to measure the signal conveniently.

7. Complete the data table. You will need two data tables, one for each type of circuit.

8. Make a plot, in Excel, of V_{Rm} and V_{Lm} as functions of frequency for the LR circuit (on a single plot), and of V_{Rm} and V_{Cm} as functions of frequency (on one more plot) for the RC circuit.

9. Make a plot of tan ϕ as a function of frequency for both cases on a single graph.

ANALYZING THE DATA

DATA TABLE

Frequency (Hz)	ω	$\mathcal{E}_m$	V_{Rm}	V_{Lm}	Phase Shift
10					

Frequency (Hz)	ω	$\mathcal{E}_m$	V_{Rm}	V_{Cm}	Phase Shift
10					

Calculations: Show all the calculations to verify the equations given in the handout.