.Experiment #3: Interference and Diffraction

EYE HAZARD: never look directly into a laser!
Starting with experiment #4,
Please always bring a formatted high-density PC diskette with you to the lab.

Purpose:

1. To study the diffraction of coherent laser light of known wavelength from a single slit.
2. To study the interference and diffraction of coherent light from a pair of closely-spaced slits.

Equipment:

Helium-Neon Laser
Optical Bench with Single-Slit and Double-Slit Apertures
Transparent Ruler
Measuring Tape

Part 1. Diffraction from a single slit

Discussion:

So far in your experiments on optics, you have been looking at phenomena (reflection, refraction, etc.) which are explained by the simple laws of geometrical optics, such as the statement that a light beam travels in a straight line in a homogeneous medium. The laws of geometrical optics are, however, only approximately true. In actual fact, light is a wave. It has a wavelength λ and a frequency f. Like sound waves, light should therefore bend around corners. This does happen, but since the wavelength of light is very small (400–700 nm for visible light) compared to the sizes of objects around us in daily life, the bending is not easily observable and the approximation of geometric optics is usually very good. But if the objects encountered by a beam of light are comparable in size to the wavelength of the light, then the effects due to the wave nature of light become observable. For example, if you illuminate a narrow slit (about 10–100 μm width) with coherent light and project its image onto a distant screen, you will not get the simple rectangular pattern you might expect. Instead, you will see alternating dark and bright bands called fringes, extending out to large lateral distances—the light has bent around the edges of the slit, in a phenomenon called diffraction. The fringes arise because light waves coming from different portions of the slit interfere constructively or destructively, depending on their path length differences from those portions to the screen.

When laser light of wavelength λ passes through a single slit of width d and then illuminates a screen a long distance beyond the slit (see diagram on next page), a characteristic single-slit diffraction pattern appears, in which the central maximum is flanked by secondary maxima on both sides. Diffraction theory predicts that *minima* of brightness (i.e., dark fringes) are produced at angles θ_m given by

$$m\lambda = d\sin\theta_m \tag{1}$$

where $m = \pm 1, \pm 2, \pm 3, \ldots$ is called the order of the minima. For very small angles, $\sin\theta_m \cong \theta_m \cong \tan\theta_m = x_m / R$ where x_m is the distance from the center of the m^{th} dark fringe on the screen to the center of the central maximum, and R is the distance from the slit to the screen. Thus, for low orders Eq. (1) becomes

$$m\lambda = \theta_m d \tag{2a}$$

or equivalently,

$$m\lambda = x_m d / R. \tag{2b}$$

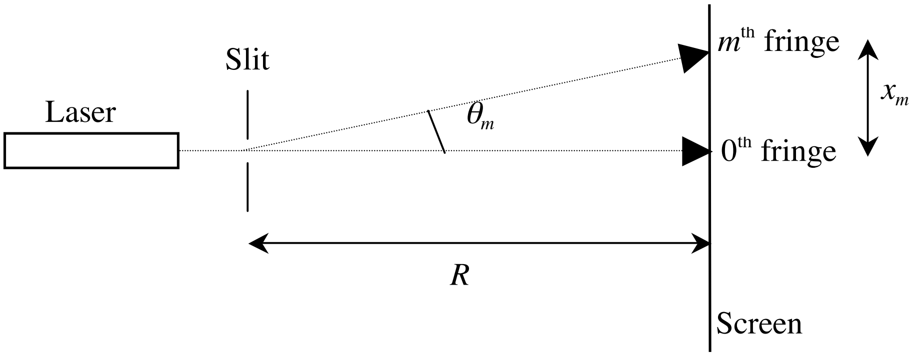
We define the width of a maximum (i.e., a bright fringe) to be half of the distance between its adjacent minima. That is,

$$\Delta x_0 = \tfrac{1}{2}(x_{+1} - x_{-1}), \;\; \Delta x_1 = \tfrac{1}{2}(x_{+2} - x_{+1}), \;\; \text{etc.} \tag{3a}$$

or equivalently,

$$\Delta \theta_0 = \tfrac{1}{2}(\theta_{+1} - \theta_{-1}), \;\; \Delta \theta_1 = \tfrac{1}{2}(\theta_{+2} - \theta_{+1}), \;\; \text{etc.} \tag{3b}$$

gives the width of the 0th (central) maximum, 1st (first secondary) maximum, etc.



Procedure:

A single-slit diffraction pattern will be projected across the room onto the board using a HeNe laser (whose wavelength is $\lambda = 632.8$ nm). Mark the centers of the $m = \pm 1, \pm 2$, and ± 3 minima. Carefully measure the following three separations:

$$\tfrac{1}{2}(x_{+1} - x_{-1}) \;\text{ and }\; \tfrac{1}{4}(x_{+2} - x_{-2}) \;\text{ and }\; \tfrac{1}{6}(x_{+3} - x_{-3}). \tag{4}$$

These values should all be nearly the same and theoretically should equal x_1. So average them to get one best value of x_1. Also carefully measure the distance R from the slit to the screen and then use Eq. (2b) to determine the slit width d. Show all calculations in your report including units. Make sure your final answer is reasonable (i.e., compare it to the discussion in the principles section above). Compute a percent error by comparing to the actual value of d written on the slit.

Supplemental Questions:

1. Use Eq. (2b) to verify theoretically that all three quantities in Eq. (4) are in fact equal to x_1.

2. Use Eqs. (2b) and (3a) to find theoretical expressions for Δx_0 and Δx_1 in terms of R, λ, and d. If the distance from the slit to the screen is doubled, what will happen to these two widths?

3. Use Eqs. (2a) and (3b) to find theoretical expressions for $\Delta\theta_0$ and $\Delta\theta_1$ in terms of λ and d. If the distance from the slit to the screen is doubled, what will happen to these two widths?

4. Use your answers to questions 2 and 3 to determine how much wider the principal maximum is than the first secondary maximum. Does the answer depend on whether you use Δx or $\Delta\theta$ for the widths? Briefly explain why or why not.

5. What will happen to the diffraction pattern if the slit width is doubled (but nothing else is changed)? Be specific and quantitative in your answer.

6. What will happen to the diffraction pattern if a blue laser (with $\lambda = 470$ nm) is used to illuminate the slits instead of the red HeNe laser (but nothing else is changed)? Be specific and quantitative in your answer.

7. If both the red and blue lasers simultaneously illuminate the slit, what pattern will be seen on the screen?

Part 2. Diffraction from two slits

Discussion:

Now suppose that we illuminate two slits with our helium-neon laser, where each slit has the same width *d* and the centers of the two slits are separated from each other by a distance *a*, where we will suppose that $a >> d$ (the >> means "much greater than"). Then, the pattern observed on a distant screen will have the following characteristics. There will be an overall "envelope" pattern of maxima and minima identical to the single-slit case. However, within each single-slit maximum there will be a number of rapidly varying and regularly spaced fringes, resulting from the interference of the light coming from the two slits. The position of the p^{th} *maximum* of this rapidly modulated interference pattern is

$$p\lambda = a\sin\theta_p \tag{5}$$

where $p = 0, \pm1, \pm2, \pm3, \ldots$ is called the order of the maximum. Making the same kind of approximations as in the single-slit case, this can also be written as

$$p\lambda = \theta_p a \tag{6a}$$

or

$$p\lambda = x_p a / R \tag{6b}$$

for low orders.

In analogy with what we did in part 1, we define the width of an interference maximum to be the distance between it and one of its adjacent minima. Since the minima are exactly midway between the maxima, we conclude that

$$\Delta x_0 = \tfrac{1}{2}(x_{+1} - x_{-1}),\ \ \Delta x_1 = \tfrac{1}{2}(x_{+2} - x_{+1}),\ \text{ etc.} \tag{7a}$$

or

$$\Delta\theta_0 = \tfrac{1}{2}(\theta_1 - \theta_0),\ \ \Delta\theta_1 = \tfrac{1}{2}(\theta_2 - \theta_1),\ \text{ etc.} \tag{7b}$$

gives the width of the 0th, 1st, etc. maximum.

Procedure:

A two-slit diffraction pattern will be projected onto the board using the HeNe laser. Make a sketch of the two-slit pattern, indicating the positions of the bright and dark fringes and the approximate brightness of the maxima. Mark the centers of the $p = \pm1$, ±2, and ±3 interference maxima. Again measure the separations given by Eq. (4) and average them to get one best value of x_1. Now use Eq. (6b) with the same values of λ and R as before to determine the slit separation *a*. Show all calculations in your report including units. Again make sure your final answer is reasonable (compare it to the discussion in the principles section above and to the slit width you obtained in part 1). Compute a percent error by comparing to the actual value of *a* written on the slide. Also mark the positions of some of the diffraction side-lobe minima. Use them to determine the width of *d* of the two slits in exactly the same way as in part 1, and again compute a percent error. Make sure you get a value for *d* which is smaller than what you got for *a*!

Supplemental Questions:

8. On your sketch of the kind of two-slit pattern, label the sets of bright and dark fringes arising from interference between the two slits with various integers p. Also label the lobes arising from diffraction from an individual slit with various integers m. If necessary, refer to your physics textbook for help.

9. Show theoretically from Eqs. (6b) and (7a) that the relation between Δx_0 and Δx_1, and $\Delta\theta_0$ and $\Delta\theta_1$ are the same. Contrast this with the answer to question 4 in part 1.

10. University Physics students only: Explain qualitatively how it is that even though Eqs. (1) and (5) appear at first glance to be identical (except for naming the symbols differently), they actually are quite different: How is it that Eq. (1) gives the minima while Eq. (5) gives the maxima? How is it that Eq. (1) skips the integer value 0, while Eq. (5) does not? A detailed mathematical analysis is not sought, just a simple discussion of these issues. Avoid giving uninspiring answers, such as "that's what the theory predicts" or "if you substitute values in, that's what you get." The point is: forgetting about equations for the moment, what is fundamentally DIFFERENT about single-slit and two-slit interference? If the spacing between two ideal slits were reduced to zero, would the two-slit pattern become a single-slit diffraction pattern?