Experiment #12: The Bohr Atom

Purpose: To observe the visible spectrum of hydrogen and helium and verify the Bohr model of the hydrogen atom.

Equipment:

Spectroscope
Hydrogen and Helium Gas Discharge Tubes, Holder, and Variac
Flashlight

Discussion:

The Bohr model of the hydrogen atom is based on the following assumptions:

(1) The electron revolves in certain preferred circular orbits around the nucleus (a single proton) at the center of the atom.

(2) The angular momentum of the electron in any of the preferred circular orbits is quantized, i.e., it can have only specific values given by

$$m\upsilon r = nh/2\pi \equiv n\hbar \qquad (1)$$

where r is the radius of the orbit, υ is the speed of the electron in the orbit, m is the mass of the electron, $h = 6.626 \times 10^{-34}$ J•s is Planck's constant, and n is a positive integer which can take on the values 1, 2, 3, ….

(3) While the electron is in a preferred orbit, its energy is a constant. The electron emits a photon if it makes a transition from an orbit of higher energy to an orbit of lower energy. It absorbs a photon if it makes a transition from an orbit of lower energy to an orbit of higher energy. The energy of the photon is equal to the difference in energies of the two orbits.

Let us see what we can deduce based on these assumptions. The only force on the electron is the Coulomb attraction by the nucleus, so from Newton's second law ($F = ma$),

$$ke^2/r^2 = m\upsilon^2/r \qquad (2)$$

where $k = 8.988 \times 10^9$ N•m^2/C^2 from Coulomb's law, and we have used the standard expression for the centripetal acceleration on the right-hand side. The total energy E of the electron in its circular orbit is given by the sum of its kinetic and potential energies as

$$E = \tfrac{1}{2}m\upsilon^2 - ke^2/r\,. \qquad (3)$$

Using Eqs. (1) and (2), it is easy to eliminate the speed υ and the radius r from the expression for the total energy to obtain

$$E_n = -\frac{2\pi^2 k^2 m e^4}{h^2}\frac{1}{n^2} = -\frac{13.6\ \text{eV}}{n^2} \qquad (4)$$

where the energy in an orbit has now been labeled by the quantum number n for that orbit. The proportionality constant was evaluated by substituting the known values of $m = 9.109 \times 10^{-31}$ kg, k, $e = 1.602 \times 10^{-19}$ C, and h, and dividing the result in J by e to get 13.6 eV. So the lowest possible energy of the electron (corresponding to $n = 1$) is –13.6 eV, the next higher energy (for

$n = 2$) is –3.4 eV, and so on.

Next let's consider the experimental setup. A diffraction grating forms a pattern of maxima and minima in the light intensity when a beam of light is passed through it, similar to the pattern formed by the double-slit arrangement which you used in Experiment #3. Recall that the location of these intensity fringes will be different for different wavelengths of light. These two facts make it possible for a diffraction grating to be used to separate the different wavelengths of light in the visible spectrum for our examination, which is known as "spectroscopy."

The light source in this experiment is a narrow tube of gas through which an electrical discharge is passed. This discharge excites electrons in the gas atoms to energy levels above the ground state; as these electrons return ("decay") to the lower states, photons will be emitted whose energies equal the differences between the relevant electron energies. The energy levels in each gas are different and the pattern of spectral lines produced by the gas are thus unique and may be used to identify the gas from which they were emitted. A detailed analysis of them can in fact reveal the energy levels and hence the nature of the constituents in the gas.

As an example, suppose the electron decays from an initial state n_i to a final state n_f of lower energy ($n_i > n_f$). The electron consequently loses energy equal to $E_i - E_f$ and a photon with the same energy is emitted. Now, the energy of a photon is equal to hf where h is Planck's constant and f is the frequency of the photon, so that

$$hf = E_i - E_f = 13.6\,\text{eV}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right). \tag{5}$$

But the frequency f speed of light $c = 2.998 \times 10^8$ m/s, and wavelength λ are related by

$$c = \lambda f \tag{6}$$

so that we can rewrite Eq. (5) as

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \tag{7}$$

where the Rydberg constant R is defined as

$$R = \frac{2\pi^2 k^2 m e^4}{h^3 c} = 1.097 \times 10^7\ \text{m}^{-1}. \tag{8}$$

Procedure:

1. Connect the hydrogen gas discharge tube to its power supply and turn it on, after verifying that the variac switch is in the 120 V position and dialed to about 80% of its maximum voltage. Mount a grating on the central platform as near perpendicularly to the collimator tube as you can, checking that the holding post is not blocking the view. Be sure the grating is mounded with its grooves oriented vertically not horizontally---check by looking through it at a ceiling lamp before mounting it! Bring the discharge tube up close to the entrance slit of the collimator of the spectroscope, moving it from side to side to get as bright an image of the zeroth-order band as possible. Next swing the viewing tube by about 20-30% in either clockwise or counter-clockwise directions and look for the first-order spectral lines. You will probably need to tilt the grating, using three hold-down screws on the central platform, to center the spectral lines vertically in the eyepiece, as otherwise they tend to be too low or too high to measure properly. First measure the angle θ_0 (in

degrees to within 0.5° and minutes to within 30' using the vernier) of the bright central (zeroth-order) maximum, by aligning the cross-hair horizontally on the center of the band of light; note that the knob near the bottom of the spectroscope can be used to fine tune the angle. You may need to open or close the slit to let more or less light in. Also, the eyepiece can be slid in or out to focus the image if necessary.

2. Next, resolve the spectral lines into individual colored bands in first order and determine the angle θ_S at which each line has been diffracted; record the angles in degrees and minutes. You should find in order of increasing angle: a dim and a bright violet line about a centimeter apart in the viewer, a blue line, and a red line. Ignore other blurry features and artifacts.

3. The angle of diffraction in first order is related to the wavelength of that particular line according to the grating equation,

(9)

where d is the spacing between adjacent grooves in the grating. The reciprocal $1/d$ is the number of grooves per unit length (usually specified in lines/mm). The value of θ, the corrected angle of diffraction, is equal to

$$\theta = \left|\theta_S - \theta_0\right|. \qquad (10)$$

Once the wavelengths are determined experimentally using two equations, we can compare them with the theoretical wavelengths calculated from Eq. (7) and identify the electron transition which results in each spectral line.

4. We will similarly measure the spectral lines of helium, although the theory for calculating its wavelengths is too advanced to review here. In the best setups, up to 12 lines have been observed (3 very dim violet lines close together, a dim violet further removed, royal blue, blue-green, 3 green lines close together with the last being ghostly in appearance, orange-yellow, and 2 reds). Measure the angles of as many as you can see in first order.

Data Analysis:
Analyze the data in Excel in four sections:

1. Color Table

First, prepare a table showing the wavelength ranges of the colors in the visible spectrum. The following cutoffs are necessarily somewhat arbitrary, but suffice for our purposes:

Color	Wavelength Range (nm)
Red	630–700
Orange	590–630
Yellow	570–590
Green	500–570
Blue	450–500
Violet	400–450

2. Theory of Hydrogen Atom

Next, calculate the wavelengths of hydrogen from Eq. (7) and confirm that only four lines lie in the visible range:

Rydberg constant R (1/m) 1.097e7

n1	Lyman Wavelengths (nf=1) (nm)	Balmer Wavelengths (nf=2) (nm)	Balmer Colors	Paschen Wavelengths (nf=3) (nm)
2	<formula>	–	–	–
3	<formula>	<formula>	<color>	–
4	<formula>	<formula>	<color>	<formula>
5	<formula>	<formula>	<color>	<formula>
6	<formula>	<formula>	<color>	<formula>
7	<formula>	<formula>	–	<formula>
8	<formula>	<formula>	–	<formula>

where <formula> = 1e9/$R/(1/nf^2–1/ni^2), the values of ni and nf are tabulated abouve, and <color> is identified from the Color Table in Sec. 1, adding modifiers such as “Light” and “Dark” if two wavelengths have the same nominal color.

3. Experimental Hydrogen Spectrum

Third, fill in the following table of the observed colors and angles for each visible line of hydrogen and use Eqs. (9) and (10) to determine *d* by matching the observed colors with the wavelengths of the nearest theoretical colors. Then compute an average *d* as explained below.

The bottom angular scale under the magnifying lens gives the number of degrees to within 0.5˚ and the upper vernier scale gives up to 30' of arc:

zero angle (rad) =radians(degrees+minutes/60)

Observed Color	Raw Angle (rad)	Corrected Angle (rad)	Theoretical Wavelength (nm)	Groove Spacing (micrometer)

Enter the observed four colors, use the same formula as for the zero angle above to compute the raw angles for each color, compute the corrected angle as the absolute value of the raw minus the zero angle, copy and “Paste Special...(values)” the four appropriate theoretical wavelengths corresponding to each observed color from Table 2 above, and finally compute the groove spacing by multiplying each theoretical wavelength multiplied by 1e–3 to convert to μm and divided by the sine of the corrected angle, in accord with Eq. (9). Below the table, type the following:

average groove spacing d (micrometer) =average(<column>)
grating ruling (lines/mm) =1e3/d

where <column> means click and drag over the four numerical values of the groove spacings above. Typical rulings range from a few hundred to a few thousand lines/mm. Make sure your answer lies in this range.

4. Experimental Helium Spectrum

Finally, make a table of observed colors and angles for each visible line of helium and compute their experimental wavelengths using Eq. (9) with the average value of *d* found above, remembering to multiply by 1e3 to convert back to nm.

Observed Color	Raw Angle (rad)	Corrected Angle (rad)	Computed Wavelength (nm)

Discuss in your results section how well the computed wavelengths agree with the observed colors by referring to your color table above.

Supplemental Questions:

1. Solve Eqs. (1) and (2) simultaneously to get expressions for r and υ in terms of n and fundamental constants only. Hence, compute the radius of the ground state orbit in Å (10^{-10} m); this is known as the Bohr radius and is usually denoted by a_0. Find the speed of the electron in this orbit expressed as a percentage of *c*, the speed of light. Also substitute your two general expressions for r and υ into Eq. (3) to verify the theoretical form of Eq. (4), i.e., the first equality.

2. Plug the values of the various constants into Eq. (4) and verify the computed value for the proportionality constant, i.e., the second equality including units. Referring to Table 2 of your Excel spreadsheet, prepare an energy level diagram to scale (in Excel preferably) of horizontal lines marking energy levels 2 through 6 only, with the spacings between the lines proportional to the energy differences between the levels. Then draw arrows to indicate the emission processes for decay from $n_i = 3, 4, 5,$ and 6 to $n_f = 2$, similar to the proportion of a figure you will find in your textbook (e.g., Fig. 4 on page 1071 of Halliday, Resnick, and Krane; Fig. 30.11 of Cutnell & Johnson). Do not photocopy a textbook figure; prepare your own showing levels 2 to 6 only and filling up most of a page so it's easy to read!

3. Verify the theoretical and numerical values of Eq. (8), i.e., verify the first equality by manipulating the appropriate equations and then verify the second equality by substituting in the appropriate numbers including units.

II. EMISSION SPECTRA 2000-10,000 A (Continued)

HELIUM

Wave length	Arc	Discharge tube	Wave Length	Arc	Discharge tube	Wave length	Arc	Discharge tube
II 2252.71	. .	10	I 3705.00	. .	30	I 5015.67	. .	100
II 2306.22	. .	20	I 3732.86	. .	10	I 5047.74	. .	15
II 2385.42	. .	30	I 3819.61	. .	50	II 5411.55	. .	50
II 2511.22	. .	50	I 3867.48	. .	15	I 5875.62	. .	1000
I 2723.19	. .	10	I 3888.65	. .	1000	I 5875.87	. .	10
II 2733.32	. .	100	I 3964.73	. .	50	II 6560.13	. .	100
I 2763.80	. .	20	I 4009.27	. .	10	I 6678.15	. .	100
I 2829.07	. .	40	I 4026.19	. .	70	I 7065.19	. .	70
I 2945.10	. .	100	I 4120.81	. .	25	I 7065.70	. .	10
I 3187.74	. .	200	I 4143.76	. .	15	I 7281.35	. .	30
II 3203.14	. .	100	I 4387.93	. .	30	I 7816.16	. .	12
I 3354.55	. .	10	I 4437.55	. .	10	I 9463.66	. .	60
I 3447.59	. .	15	I 4471.48	. .	100	I 9516.70	. .	30
I 3587.25	. .	10	II 4685.75	. .	300	I 9526.17	. .	10
I 3613.64	. .	30	I 4713.14	. .	40	I 9702.76	. .	10
I 3634.23	. .	15	I 4921.93	. .	50			

HOLMIUM

Wavelength	Arc	Spark	Wavelength	Arc	Spark	Wavelength	Arc	Spark
2431.03	. .	20	3281.98	12	15	3837.45	15	6 n
2433.00	. .	20	3289.38	10	20	3854.05	10	20
2439.33	. .	20	3338.76	12	20	3861.68	40	20
2442.76	. .	20	3343.56	20	20	3888.95	40	20
2452.69	. .	20	3372.79	12	15	3891.02	200	40
2511.12	. .	20	3398.98	40	60	3905.55	15	8
2597.51	. .	20	3410.25	20	15	3905.78	30	6
2636.50	. .	70	3414.92	30	30	3955.74	15	4
2677.95	. .	20	3416.46	30	40	3998.28	40	6
2681.18	. .	20	3421.64	20	20	4040.84	150	30
2774.70	. .	300	3425.35	40	40	4045.43	200	80
2812.87	. .	20	3428.13	40	40	4053.92	400	200
2814.74	10	20	3429.19	10	15	4101.09	40	40
2824.19	20	3	3453.13	30	20	4103.84	400	400
2826.63	3	20 n	3456.00	60	60	4108.63	100	40
2828.14	. .	20 n	3461.96	20	20	4120.20	50	25
2831.60	. .	70	3474.25	40	20	4125.65	20	15
2845.64	. .	70 n	3484.73	40	30	4127.16	150	60
2847.50	. .	40	3494.77	30	40	4136.24	40	25
2849.10	10	20	3515.58	40	40	4152.54	30	30
2867.82	. .	40 n	3531.74	10	20	4152.75	30	40
2880.27	20	10	3545.97	20	20	4163.03	100	100
2880.99	20	10	3556.76	15	40	4173.23	50	. .
2894.99	20	10	3574.78	10	20	4194.34	30	15
2897.36	. .	20 n	3598.77	40	30	4254.43	100	20
2909.42	40	10	3626.70	20	15	4264.07	15	8
2928.79	. .	100	3627.18	15	15	4350.73	40	15
2936.77	. .	1000 r	3662.27	20	10	5468.46	20	. .