

## Lab #8 Rotational Motion

**Purpose:** The following procedures will examine rotational motion from two different aspects:

- a. Variable torque with a constant moment of inertia
- b. Constant torque with a variable moment of inertia

**Equipment:** Rotational Motion Apparatus  
String + smart pulley  
Computer + Science Workshop 750  
Weight set/hanger

**Discussion:**

Essentially all of the relations between physical quantities associated with linear motion that we saw in the laboratory on Newton's 2<sup>nd</sup> Law have analogous counterparts in the realm of rotational motion. The element that links the two types of motion is an object's distance from the axis of rotation, or spin axis, and this radius is usually denoted by  $r$ .

<u>Linear variable</u>		<u>(units)</u>	<u>Rotational analog</u>		<u>(units)</u>
Displacement	$x$	(m)	angular displacement	$\theta$	(rad)
Velocity	$v$	(m/s)	angular velocity	$\omega$	(rad/sec)
Acceleration	$a$	(m/s/s)	angular acceleration	$\alpha$	(rad/sec <sup>2</sup> )

The fact is that  $x = r\theta$ ,  $v = r\omega$ , and  $a = r\alpha$ .

(Also just as  $v = \frac{\Delta x}{\Delta t}$  and  $a = \frac{\Delta v}{\Delta t}$ , so  $\omega = \frac{\Delta \theta}{\Delta t}$  and  $\alpha = \frac{\Delta \omega}{\Delta t}$ .)

Suppose we multiply both sides of the Newton's 2<sup>nd</sup> Law equation by  $r$  in search of some sort of similar relation for rotational motion:

$$F \cdot r = m \cdot a \cdot r$$

But  $F \cdot r = \tau$  (torque), and since  $a = r \cdot \alpha$ , we have:

$$\tau = (m \cdot r^2) \cdot \alpha \tag{1}$$

The expression in parentheses in equation (1) represents the moment of inertia,  $I$ , and has units of  $\text{kg} \cdot \text{m}^2$  in the metric system. Moment of inertia is to rotational motion what mass is to linear motion. (Similarly, torque in rotational motion is analogous to force in linear motion.) It turns out that just as linear acceleration is directly proportional to force and inversely proportional to mass, angular acceleration is directly proportional to torque and inversely proportional to moment of inertia.

Moment of inertia can be defined as a measure of a body's resistance to changes in its rotational motion. It considers not only how much mass the body has, but also how far that mass is located from the axis of rotation. The full roll of paper towels is more difficult to rotate than the almost-empty roll, not only because it has more mass, but also because that mass is located farther away from the spin axis.

To further understand the concept of moment of inertia, consider two cylinders of identical mass but different shape:

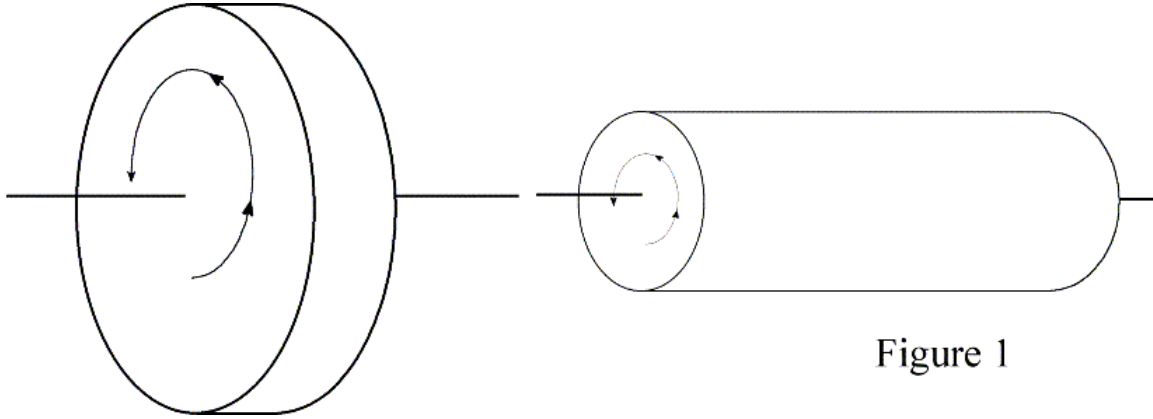


Figure 1

If both are initially at rest and we put the same constant torque on each axle, which one would experience the greater rotational acceleration? In other words, which one would achieve an angular velocity of, say, 10 rad/sec sooner? Answer: the one on the right, because ITS moment of inertia is smaller since its mass is located closer to the spin axis. (Since  $\tau = I * \alpha$ , and torque is constant, if I is less,  $\alpha$  must be greater.)

In addition to mass and distance of the mass from the spin axis, the location of the spin axis itself affects moment of inertia. For example, it's easier to rotate a meter stick held in the middle than one held at one end; one can feel this difference in one's wrist. More torque is required to accelerate the meter stick held at one end at the same rate as one held in the middle.

## PRELIMINARY INFORMATION

$$\tau = I \cdot \alpha$$

Newton's 2<sup>nd</sup> Law lab: Part I: increasing driving FORCE & constant system MASS.  
(Result.....increasing linear acceleration.)  
Now, rotational analogy: Part I: increasing TORQUE & const. MOMENT OF INERTIA.  
(Result.....increasing angular acceleration.)  
Newton's 2<sup>nd</sup> Law lab: Part II: increasing system MASS & constant driving FORCE.  
(Result.....decreasing linear acceleration.)  
Now, rotational analogy: Part II: increasing MOMENT OF INERTIA & const. TORQUE  
(Result.....decreasing angular acceleration.)

The mass hanging from the end of the string provides the tension in the string, which rotates the platform. If the tension in the string is T when the apparatus is rotating, then the torque exerted by it is:

$$\tau = r \cdot T \tag{1}$$

where r is the radius of the cylinder about which the string is wound (1.5 cm). The two forces on the hanging mass are the upward tension in the string and its weight, mg, so if a is the acceleration of the hanging mass, then by Newton's 2<sup>nd</sup> Law,

$$mg - T = m a \tag{2}$$

Solving for the tension in the string yields:

$$T = m (g - a) \tag{3}$$

So the torque = m (g - a) r (4)

You will use the Data Studio icon, Lab 02/03, and Newton's Law (smart pulley) options with the computer to record the LINEAR acceleration of the falling mass, just as you did in the two Newton's 2<sup>nd</sup> Law labs. Once you have the linear acceleration, you can easily calculate the ANGULAR acceleration of the rotating platform using the relation  $a = r \alpha$ . Again, the radius here is that of the aluminum cylinder around which the string wraps.

In Part I, you'll construct a graph of torque versus angular acceleration, the slope of which should equal a constant moment of inertia of the platform (analogous to the plot of F vs. a that you did in Part I of the Newton's 2<sup>nd</sup> Law lab, the slope of which was the constant system mass).

In Part II, you'll have a constant amount of mass on the end of the string each trial, but you'll change the moment of inertia of the platform by attaching a pair of 275-g masses at different points. You'll construct a graph of moment of inertia versus the reciprocal of angular acceleration, the slope of which should equal the constant torque on the platform (analogous to the plot of M vs. 1/a that you did in Part II of the Newton's 2<sup>nd</sup> Law lab, the slope of which was the constant driving force).

**Procedure Part One:**

1. The first thing you should do is be sure the platform is level. (Your instructor will show you how to do this.) After leveling, begin with both stop-knobs centered at the 24-cm points and nothing else on the platform.
2. Beginning with a TOTAL mass of 100 grams on the end of the string, wind the platform so that the hook of the mass hanger is just below the pulley. Ensure the hanging mass is now swinging, and start the elapsed time clock with a click of your mouse. Then release the platform and let the hanging mass accelerate toward the floor.
3. Repeat this process 4 more times, each time increasing the hanging mass by 50-grams increments, so the last trial will use 300-grams, include hanger.
4. After using Excel to help you calculate torque and angular acceleration values for each run, plot torque vs. angular acceleration. The slope of the trendline is your experimental value for the moment of inertia of the rotating platform. The actual value is  $0.0126 \text{ kg m}^2$ . Calculate percent error.

Data table for Part One should look like this:

Trial No.	Driving Mass (kg)	1/2 a (m/s <sup>2</sup> )	a (m/s <sup>2</sup> )	Alpha (rad/s <sup>2</sup> )	Torque (N m)
1	0.1				
2	0.15				
3	0.2				
4	0.25				
5	0.3				

**Procedure Part Two:**

1. Each of your FOUR trials will use a driving mass of 250 grams (incl. hanger). With the stop-knobs still at the 24-cm points, tighten two of the black square (275-g) masses at the 5 cm points on the rotating platform. Conduct the run as in Part I.
2. Conduct three similar trials with the 275-g masses at the 10 cm, 15 cm, & 20 cm points. Record the accelerations.
3. Calculate the moment of inertia for each trial by ADDING to the value provided to you by your instructor in Part I the additional moment of inertia due to the two 275-g masses as if they are point masses, using the general formula  $I = m \cdot r^2$  for each of the two masses.
4. After calculating ANGULAR acceleration for each run (just as was done in Part I), plot a final graph of moment of inertia vs. the reciprocal of angular acceleration. The slope equals the constant torque.

5. compare your experimental torque (slope of graph) with the actual average net torque, and calculate percent error. It has been determined that the actual average net torque for this lab is 0.0367 N m. (Theoretical).

Data table for Part Two should look like this:

Trial No.	Location (m)	1/2 a (m/s <sup>2</sup> )	a (m/s <sup>2</sup> )	alpha (rad/s <sup>2</sup> )	1/alpha (s <sup>2</sup> /rad)	2 m r <sup>2</sup> (kg * m <sup>2</sup> )	I Mom. of Inertia (kg * m <sup>2</sup> )
1	0.05						
2	0.1						
3	0.15						
4	0.2						