Lab #6 Collisions

Purpose: To gain an understanding of (a) how momentum is conserved in inelastic collisions, and (b) how both momentum and kinetic energy are conserved in elastic collisions.

Equipment: Paper and pencil. This is an "academic" laboratory exercise. Partners are neither required nor desired. (Do your OWN work.) The instructor will discuss physics principles involved, and will present and solve several types of problems. The student will then answer assigned questions and solve assigned problems. Neatly presented question answers and problem solutions will constitute the lab report.

Discussion: Part 1, Inelastic Collisions
If a single object experiences zero net force, then, according to Newton's Second Law, the time rate of change of momentum (the product of mass and velocity) is zero:

$$F_{net} = \frac{\Delta p}{\Delta t} = 0$$

Note that $$\frac{\Delta p}{\Delta t} = \frac{\Delta(m\,v)}{\Delta t} = \frac{m\;\;\Delta V}{\Delta t} = m\;a = F_{net} = 0$$

(Since m does not = zero, a and F must both be zero.)

This is the same as saying that an object's momentum remains constant if the net force acting on it equals zero, or:

$$p = m\,v = \text{constant}$$

To date we have looked at forces acting on a single object. Now we will consider the motion of a <u>system</u> of interacting objects (collisions). For such systems, Newton's laws apply to each individual object, so that while the momenta of the individual objects may change, the total momentum of the <u>system</u> is constant whenever the net external force on the system = 0. The effect of the internal forces of interaction among the two objects is to exchange momentum between them in such a way that the total momentum is conserved (constant). Another way of stating the Law of Conservation of Linear (as opposed to angular) Momentum is: If no external forces (friction, for example) act on a system, the total momentum before the collision is equal to the total momentum after the collision.

Momentum is a vector quantity, and we usually consider objects in motion to the right as having positive momenta in conformity with the Cartesian coordinate system. For a collision between two objects, the Law of Conservation of Linear Momentum can be expressed algebraically as:

$$m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2' \qquad (1)$$

The subscripts indicate to which of the two bodies we are referring, and the unprimed quantities stand for values before the collision while primed quantities stand for values after the collision. This law holds for any collision on any scale, from subatomic to galactic.

Inelastic collisions are those in which some kinetic energy is lost. They are "real world" collisions, and there are two types: ordinary inelastic collisions, in which the two objects "bounce off" each other, and obey equation (1) above, and perfectly inelastic collisions, after which the two end up "stuck together" and (usually) move as if one larger object. In the latter case, v_1' and v_2' are the same (call it V'), so equation (1) above becomes:

$$m_1 V_1 + m_2 V_2 = (m_1 + m_2) V' \qquad (2)$$

or

$$V' = \frac{m_1}{m_1 + m_2} v_1 + \frac{m_2}{m_1 + m_2} v_2 \qquad (3)$$

So if we know the masses and initial velocities in a perfectly inelastic collision, we can find the final velocity from equations (2) or (3). But if the two objects do not stick together, and usually have different velocities after collisions, then we have two unknowns (the two final velocities). We would need to be told one of these two after-collision velocities in order to find the other using equation (1).

One practical application of the perfectly inelastic collision involves the ballistic pendulum (See Figure 1).

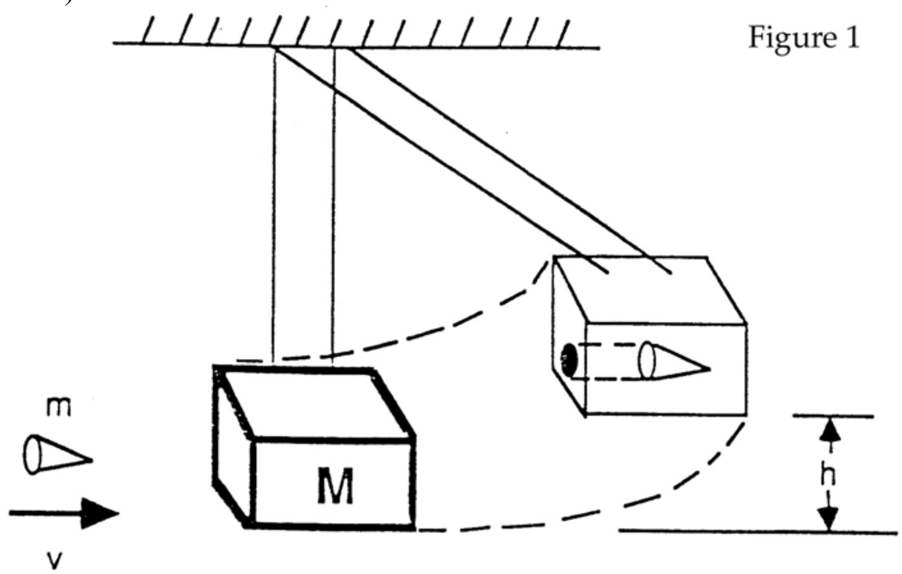

Figure 1

A projectile is filed into a wooden block that is free to swing back, and the height to which the block rises can be used to find the initial speed of the projectile. A ratchet assembly keeps the block from swinging back down after it achieves its maximum height (by permitting motion in only one direction). The bullet of mass m has a velocity v and the block of mass M is at rest initially. The bullet actually comes to rest in the block before the block moves appreciably, so there are no external forces on the system and momentum is conserved. Let V be the initial velocity of the block immediately after the collision.

Then:

$$m\,v = (m + M)V \qquad (4)$$

Just after the collision, the bullet/block combination have a kinetic energy as follows:

KE = _(m + M)V^2

After the block has swung back to its highest point, this initial KINETIC energy has all been converted to gravitational potential energy equal to (m + M)gh. By equating these two energy expressions, we can solve for the bullet velocity v:

_(m + M)V^2 = (m +M)gh

So

$$V = \sqrt{2gh}$$

Inserting this in equation (4) we have:

$$v_{Bullet} = \frac{m + M}{m}\sqrt{2gh}$$

Next, consider a hard ball of mass m dropped from rest from a height h, that rebounds to a height offset from vertical for clarity. Its total energy prior to release is all potential (mgh) and its total energy just before impact with the floor is all kinetic (1/2 mv^2). Assuming no air resistance, equating these two using the Law of Conservation of Mechanical Energy, we have:

$$mgh = 1/2\, mv^2$$

or

$$v = \sqrt{2gh} \qquad (5)$$

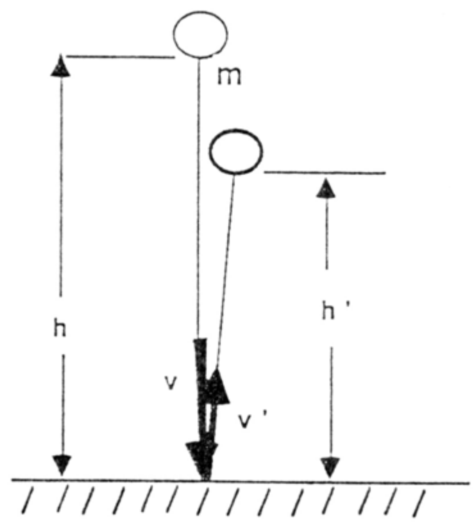

Figure 2

Similar analysis for the upward path after the collision with the floor provides the speed just after impact, v’, in terms of the maximum rebound height, h’:

$$v' = \sqrt{2gh'} \qquad (6)$$

Combining equations (5) and (6) yields:

$$\frac{v'}{v} = \sqrt{\frac{h'}{h}} \quad \text{or} \quad \frac{h'}{h} = \left(\frac{v'}{v}\right)^2$$

If the ball were to rebound to its initial height (h’=h) then we would call the collision <u>elastic</u>, because there would be NO loss of kinetic energy (which is proportional to v^2). If the ball does NOT reach its initial height on the rebound, then some kinetic energy was lost and the collision is therefore inelastic.

Elastic collisions:Suppose two objects moving horizontally experience an elastic collision. Since by definition there is no loss of kinetic energy.

$$KE_{Before} = KE_{After}$$

$$1/2m_1v_1^2 + 1/2m_2v_2^2 = 1/2m_1(v_1')^2 + 1/2m_2(v_2')^2 \qquad (7)$$

Again, the subscripts refer to which object, and the prime notation refers to after-the-collision velocities. (Be careful not to confuse the prime notation with exponents.) Dividing equation (7) by _ yields:

$$m_1v_1^2 + m_2v_2^2 = m_1(v_1')^2 + m_2(v_2')^2 \qquad (8)$$

With equation (8), we now have a second equation (in addition to equation (1)) that we can use to solve for the two unknowns mentioned earlier, the two final velocities.

Remember:
Momentum is always conserved in all collisions. If kinetic energy is also conserved, then the collision is elastic. If some KE is lost, it is an inelastic collision. If KE loss is a maximum (2 objects stick together), then it's perfectly inelastic collision.

Conservation of Linear Momentum—Practice Problems

(Take notes on solutions)

1. Two gliders approach each other on an air track. The 300-g glider is moving to the right at 2 m/s, while the 600-g glider is moving to the left at 4 m/s. If the velocity of the 300-g glider after the collision is 8 m/s to the left, find the velocity of the 600-g glider after the collision, as well as its direction.

2. A 750-kg Yugo and a 1750-kg Lincoln Continental have a head-on collision on Nine Mile Road. (No one gets hurt seriously because they were all buckled up.) The Yugo was headed west at 45 mph and the Lincoln was headed east at 30 mph. If the collision was perfectly inelastic, in what direction and at what speed did the resulting combination travel immediately after impact. Assume no friction.

3. A small explosive charge separates two sections of a stationary satellite in frictionless outer space. If the 100-kg smaller section's resulting velocity is 3 m/s, what is the resulting velocity of the 350-kg larger section? Also, which section has the greater kinetic energy?

4. A marble is shot at another marble that is at rest. The first marble stops, and the second marble continues in a straight line with the same speed that first marble had initially. What is the ratio of the masses of the two marbles?

5. A 4-g bullet from a .22 rifle is fired into the 1.2-kg wooden block of a ballistic pendulum, causing it to swing back and rise vertically 60 cm. Find the speed of the bullet just before impact.

6. A 1.00-kg steel ball is dropped from a height of 2.00 m, and it reaches a max height of 1.5 m after the 1^{st} bounce. How much of its kinetic energy did it lose in the collision with the floor?

7. Two objects moving in a straight line toward each other undergo an <u>elastic</u> collision. The one approaching from the left has a mass of 2-kg and a velocity of 6 m/s, while the other has a mass of 4-kg and a velocity of 3 m/s. What are the speeds and directions of the objects after the collisions?