Lab #1 Familiarization and The meaning of measurement and errors

Purpose: To gain an understanding of measurement and the rudiments of correct handling of significant figures and uncertainties.

Equipment:

Three solids of different shapes: Block, Cylinder, and Sphere
Ruler or meter stick
Vernier caliper
Micrometer caliper

Discussion:

What scientists discover about the world around us comes from measurement and the quantities calculated from those measurements. For this reason, you must learn to treat measured values with respect and in such a way that you extract the maximum information from those measurements without inferring more than is justified by the measured values. You should also be aware of the errors in your measurements and report them honestly. For example, if you say that the length of a table is 78.3 cm, you never really mean that the length is exactly 78.3 cm. There will be an error, or uncertainty in your measurement, which could be as small as the smallest division on your meter stick, that is 1 mm, or 0.1 cm. You would then report that the length of the table is 78.3 cm ± 0.1 cm. In some cases, as you will see, the error may be larger than the smallest division on the meter stick. Then you report that, and say that the length is 78.3 cm ± 0.4 cm, or whatever your actual error may be. In this lab, you will learn the methods for estimating the errors in your measurements. To see the significance of these remarks, we will make a few simple measurements.

Procedure:

1. Use a meter stick and measure the dimensions of the three solids. For the sphere, measure the diameter. For the cylinder, measure the diameter and the height. For the block, measure the length, width, and height. Measure each quantity 5 times and record your measurements in tables in Excel. A sample data table is shown below. You should make similar tables for the cylinder and block.

Diameter of the sphere

Trial No	Diameter (cm)	Deviation from avg (cm)	Sgr of dev (cm^2)
1	3.4	-0.02	0.0004
2	3.3	-0.12	0.0144
3	3.2	-0.22	0.0484
4	3.5	0.08	0.0064
5	3.7	0.28	0.0784
Average	3.42	5.32907E-16	0.030
Stdev	0.17		

Description of the table:
At the top is a heading describing the data. The first column is the trial number (Trial No). In this case you have made 5 repeated measurements of the same quantity. The second column (Diameter) gives the measured values. Note the units given in parenthesis beside the name of the measured quantity. That must always be specified. At the bottom of this column, Excel can calculate the average value of the diameter, which is simply the sum of the five values above, divided by the number of trials, which in this case, is 5. The average value, in this case, 3.42 cm (NOT just 3.42) will be the value that you will report as the diameter of the sphere. The questions that arise here are, how many significant figures should we report, what is the error in this measurement, and how do we estimate it?

First, notice that all the measured values are not the same. In general, they never are the same, if the experimenter is honest. In the third column (Deviation from avg), we tabulate the difference between the average value, and each measured value from the previous column. You don't have to calculate each one of these deviations by yourself. Excel can do it for you. But you should know what they mean.

Note that we found the average of these deviations at the bottom of that column. That is by definition zero, since there are as many positive deviations as there are negative deviations. Excel has given, as you can see, an absurdly small number, which is essentially zero.

In the last column, we calculate the square of the deviations. At the bottom of that column, we find the average of the squares of the deviations. This is not zero, since all the squares are positive.

At the bottom of the table, we calculate the square foot of the average of the squares of the deviations. This is called the standard deviation, abbreviated stdev. This is our error, also known as the uncertainty in the diameter of the sphere.

Note that in each case, Excel or any calculator can give an enormous number of decimal places. We will now see how to deal with these.

Since we are using a meter stick, on which the smallest division is 1 mm, or 0.1 cm, none of the values in the Diameter column, when entered from the measurement, should have more than one decimal place (when reporting in cm). If we put in an extra decimal place, we are cheating! When calculating averages or squares or square roots, we retain an extra decimal place till the end. Our standard deviation or error or uncertainty in the measurement turns out to be 0.17 cm, which is a little more than the smallest division on our scale.

So now we report the result of our measurement to be

The diameter of the sphere is 3.42 cm ± 0.17 cm. The result shows that there is an error in the first decimal place, so the second decimal is not of significance. The report should then give only two significant figures in the diameter, and in the uncertainty, and say that

The diameter of the sphere is 3.4 cm ± 0.17 cm.

AVERAGE and STDEV functions in Excel:
We calculated the standard deviation in several stops in the example above to be clear about the meaning of this quantity. You need not go through these steps every time. In one step, Excel can give you the average (use the AVERAGE() function) and in one more step Excel can also calculate the standard deviation (use the STDEV() function). Check the value that Excel gives when you use the STDEV() function. It will be slightly different from what we got. (University Physics students- find out why)

Note on significant figures:
Our measurements reported above are shown to two significant figures. Note that significant figures are different from decimal places. The same measurement of 3.4 cm can also be written as 0.034 m, or 0.000034 km, but the number of significant figures is still two. In fact, we saw in our error estimation above that the error was in second significant figure, not in the first decimal place. The decimal place is arbitrary depending on the units. The significant figures are not.

Do not carry more significant figures in a final result than are represented in the measurements. The temptation to do this is great if you are using a calculator that carries many digits. After you have completed all your calculations, you must discard all those digits that exceed the level of significance allowed by the measurements. (University Physics students, for more discussion and examples, refer to your textbook, Ch 1).

Except for discrete values such as pure counting, every measurement will have finite precision and, therefore, some level of uncertainty. This uncertainty in a measurement can be estimated in various ways and usually conforms to the number of significant figures in the measurement. The uncertainty in the measured value will lead to every measured value should be accompanied by a record of the uncertainty in that value and all computed quantities should be accompanied by an estimate of the resulting uncertainties.

Often in the laboratory a "handbook value" is available with which to compare values determined from the measurements. The acceleration of gravity, "g", is an example. In such cases, you can find the difference between your reported value and the known value. But often, and almost always in the real world, the only information about the values inferred from the measurements comes from the measurements themselves. Each individual measurement can then be compared only with the average value of your own measurements, which is what we did above. In these situations it is crucial that you understand the correct handling of measured values.

Estimating Uncertainties in Computed Values From Uncertain Measurements:
Suppose you measured the diameter, D, of the sphere with uncertainty, ΔD. The question now is, what is the error or uncertainty in the volume V of the sphere? The volume of the sphere written in terms of the diameter is given by the formula:

$$V=\frac{\pi}{6}D^3$$

With this relationship between V and D we have, from calculus, that a uncertainty ΔD in D will result in an uncertainty ΔV in V given by

$$\Delta V = \frac{\pi}{6}3D^2\,\Delta D=\frac{\pi}{2}\,D^2\,\Delta D$$ (University Physics students, verify this)

The value of D to be plugged into the above formula is the average value of D, and ΔD is the uncertainty in D.

You may now state in your report that the volume of the sphere

Volume = $V \pm \Delta V$.

This is your estimate of the volume from the measurement and your estimate of the uncertainty in the volume from the uncertainty in the measurement.

Relative uncertainty:
Sometimes it is desirable to write the relative uncertainty in the volume, $\Delta V/V$, in terms of the relative uncertainty in the measurement, $\Delta D/D$. The result has more meaning and is often simpler in form. For our sphere example, we have after some simplification

$$\frac{\Delta V}{V} = 3\frac{\Delta D}{D}$$ (University Physics students, verify this)

In this case, the relative uncertainty in V is three times the relative uncertainty in D. If D is uncertain by 10% then V would be uncertain by 30%.

For the block, with dimensions $x \pm \Delta x$, $y \pm \Delta y$, and $z \pm \Delta z$, the volume is

$$V = x\,y\,z$$

with uncertainty

$$\Delta V = |\,\Delta x\,y\,z\,| + |\,x\,\Delta y\,z\,| + |\,x\,y\,\Delta z|$$

Here we added the contribution to the error from each quantity. The errors are added as absolute values since errors do not subtract.

The relative uncertainty is

$$\frac{\Delta V}{V} = \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}$$

For the cylinder, with height $h \pm \Delta h$, and diameter $D \pm \Delta D$, the formula for the volume is

$$V = \frac{\pi}{4} D^2 h.$$

The uncertainty in the volume is given in terms of the uncertainties in height and diameter as

The relative uncertainty is

$$\frac{V\Delta}{V} = 2\frac{\Delta D}{D} + \frac{\Delta h}{h}$$

Procedure continued…

2. Note that when you make the Excel tables for the cylinder and the block, you will have more than one column of measurements. In the case of the sphere, you had just one column for the diameter. For the cylinder, you will have two columns, one for the diameter, and one for the height of the cylinder. For the block, you will have three columns, for the length, width, and height. For each dimension column, you can get Excel to find the average and the standard deviation, and then use these formulas given above to properly report the volume and the uncertainty in the volume of each object. Make sure that in each case you get Excel to calculate the relative uncertainties and report them also.

3. Now make the same measurements on all three objects with more precise instruments, the vernier calipers and the micrometer. In each case, go through the entire process of estimation the errors, and reporting your measurements with the uncertainties. If you like, you can add more columns to your original tables, or make fresh tables for each measuring instrument.

Questions:

1. Suppose you were given a second block so that the volume of this block were, say, twice that of the one you used for this experiment. How would the relative uncertainty of the volume of the larger block compare with that of the smaller block? If you were given a third block where each dimension, X, Y, and Z were twice that of the first block, how would the relative uncertainty in the volume compare?

2. Are the three measurements of the volume of the sphere using the three different measuring instruments consistent with each other?

3. Define accuracy. Define precision. How are the two related? How do they differ? In the context of this lab, what would you say about the precision and accuracy of your measurements?