AMC 10 & AMC 12

Each year, the University of West Florida will host area high school students who will join thousands of other students all across the country to participate in the American Mathematics Competitions - AMC 10 and AMC 12 – sponsored by the Mathematical Association of America. The AMC 12 has been offered since 1950 and the AMC 10 has been offered since 1998.

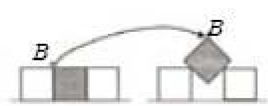
High School Students: Try these sample problems and register for the Competition!

2005 AMC 10 AND AMC 12 Sample Problems:

10#9. Three tiles are marked X and two other tiles are marked O. The five tiles are randomly arranged in a row. What is the probability that the arrangement reads XOXOX?

(A) $\frac{1}{12}$ (B) $\frac{1}{10}$ (C) $\frac{1}{6}$ (D) $\frac{1}{4}$ (E) $\frac{1}{3}$

10#19. Three one-inch squares are placed with their bases on a line. The center square is lifted out and rotated 45°, as shown. Then it is centered and lowered into its original location until it touches both of the adjoining squares. How many inches is the point *B* from the line on which the bases of the original squares were placed?



(A) 1 (B) $\sqrt{2}$ (C) 3/2 (D) $\sqrt{2} + \frac{1}{2}$ (E) 2

10#3/12#2. The equations 2x + 7 = 3 and bx – 10 = –2 have the same solution x. What is the value of b?

(A) –8 (B) –4 (C) –2 (D) 4 (E) 8

10#5/12#6. A store normally sells windows at $100 each. This week the store is offering one free window for each purchase of four. Dave needs seven windows and Doug needs eight windows. How many dollars will they save if the purchase the windows together rather than separately?

(A) 100 (B) 200 (C) 300 (D) 400 (E) 500

12#12. A line passes through A(1,1) and B(100,1000). How many other points with integer coordinates are on the line and strictly between A and B?

(A) 0 (B) 2 (C) 3 (D) 8 (E) 9

12#19. A faulty car odometer proceeds from digit 3 to digit 5, always skipping the digit 4, regardless of position. For example, after traveling one mile the odometer changed from 000039 to 000050. If the odometer now reads 002005, how many miles has the car actually traveled?

(A) 1404 (B) 1462 (C) 1604 (D) 1605 (E) 1804

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Answers
10: #9. (B); #19. (D)
10/12: 10#3/12#2. (B); 10#5/12#6. (A)
12: #12. (D); #19. (B)

2004 AMC 10 AND AMC 12 Sample Problems:

12#3. For how many ordered pairs of positive integers (x,y) is x + 2y = 100?

(A) 33 (B) 49 (C) 50 (D) 99 (E) 100

12#16. The set of all real numbers x for which

$$\log_{2004} (\log_{2003} (\log_{2002} (\log_{2001} x)))$$

is defined is {x | x > c}. What is the value of c?

(A) 0 (B) 2001^{2002} (C) 2002^{2003} (D) 2003^{2004} (E) $2001^{2002^{2003}}$

10#11/12#9. A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased?

(A) 10 (B) 25 (C) 36 (D) 50 (E) 60

10#14/12#11. The average value of all the pennies, nickels, dimes, and quarters in Paula's purse is 20 cents. If she had one more quarter, the average value would be 21 cents. How many dimes does she have in her purse?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

10#7. A grocer stacks oranges in a pyramid-like stack whose rectangular base is 5 oranges by 8 oranges. Each orange above the first level rests in a pocket formed by four oranges in the level below. The stack is completed by a single row of oranges. How many oranges are in the stack?

(A) 96 (B) 98 (C) 100 (D) 101 (E) 134

10#18. A sequence of three real numbers forms an arithmetic progression with a first term of 9. If 2 is added to the second term and 20 is added to the third term, the three resulting numbers form a geometric progression. What is the smallest possible value for the third term of the geometric progression?

(A) 1 (B) 4 (C) 36 (D) 49 (E) 81

Answers
12: #3. (B); #16. (B)
10/12: 10#11/12#9. (C); 10#14/12#11. (A)
10: #7. (C); #18. (A)

2003 AMC 10 Sample Problems:

10#4. It takes Mary 30 minutes to walk uphill 1 km from her home to school, but it takes her only 10 minutes to walk from school to home along the same route. What is her average speed, in km/hr, for the round trip?

(A) 3 (B) 3.125 (C) 3.5 (D) 4 (E) 4.5

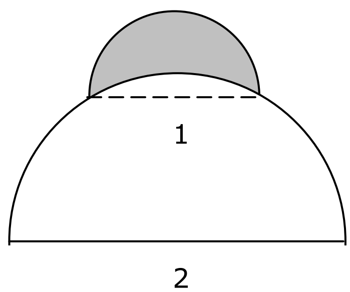
10#13. The sum of three numbers is 20. The first is 4 times the sum of the other two. The second is seven times the third. What is the product of all three?

(A) 28 (B) 40 (C) 100 (D) 400 (E) 800

10#15. What is the probability that an integer in the set {1,2,3,…,100} is divisible by 2 and not divisible by 3?

(A) $\frac{1}{6}$ (B) $\frac{33}{100}$ (C) $\frac{17}{50}$ (D) $\frac{1}{2}$ (E) $\frac{18}{25}$

10#19. A semicircle of diameter 1 sits at the top of a semicircle of diameter 2, as shown. The shaded area inside the smaller semicircle and outside the larger semicircle is called a *lune*. Determine the area of this lune.



(A) $\frac{1}{6}\pi - \frac{\sqrt{3}}{4}$ (B) $\frac{\sqrt{3}}{4} - \frac{1}{12}\pi$ (C) $\frac{\sqrt{3}}{4} - \frac{1}{24}\pi$ (D) $\frac{\sqrt{3}}{4} + \frac{1}{24}\pi$ (E) $\frac{\sqrt{3}}{4} + \frac{1}{12}\pi$

10#25. Let n be a 5-digit number, and let q and r be the quotient and remainder, respectively, when n is divided by 100. For how many values of n is q + r divisible by 11?

(A) 8180 (B) 8181 (C) 8182 (D) 9000 (E) 9090

Answers
10: #4. (A); #13. (A); #15. (C); #19. (C); #25. (B)

2003 AMC 12 Sample Problems:

12#3. A solid box is 15 cm by 10 cm by 8 cm. A new solid is formed by removing a cube 3 cm on a side from each corner of the box. What percent of the original volume is removed?

(A) 4.5 (B) 9 (C) 12 (D) 18 (E) 24

12#7. How many non-congruent triangles with perimeter 7 have integer side lengths?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

12#23. How many perfect squares are divisors of the product $1!\cdot 2!\cdot 3!\cdots 9!$?

(A) 504 (B) 672 (C) 864 (D) 936 (E) 1008

12#24. If $a \geq b > 1$, what is the largest possible value of $\log_a$ $\log_a\left(\frac{a}{b}\right)+\log_b\left(\frac{b}{a}\right)$?

(A) −2 (B) 0 (C) 2 (D) 3 (E) 4

12#25. Let $f(x) = \sqrt{ax^2+bx}$. For how many real values of a is there at least one positive value of b for which the domain of f and the range of f are the same set?

(A) 0 (B) 1 (C) 2 (D) 3 (E) infinitely many

Answers
12: #3. (D); #7. (B); #23. (B); #24. (B); #25. (C)

2001 AMC 10 Sample Problems:

4. What is the maximum number for the possible points of intersection of a circle and a triangle?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

7. When the decimal point of a certain positive decimal number is moved four places to the right, the new number is four times the reciprocal of the original number. What is the original number?

(A) 0.0002 (B) 0.002 (C) 0.02 (D) 0.2 (E) 2

10. If x, y, and z are positive with xy = 24 and xz = 48 and yz = 72, then x + y + z is

(A) 18 (B) 19 (C) 20 (D) 22 (E) 24

12. Suppose that n is the product of three consecutive integers and that n is divisible by 7. Which of the following is not necessarily a divisor of n?

(A) 6 (B) 14 (C) 21 (D) 28 (E) 42

20. A regular octagon is formed by cutting an isosceles right triangle from each of the corners of a square with sides of length 2000. What is the length of each side of the octagon?

(A) 2000 (B) $2000(\sqrt{2} - 1)$ (C) $2000(2 - \sqrt{2})$ (D) 1000 (E) $1000\sqrt{2}$

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Answers
10: #4. (E); #7. (C); #10. (D); #12. (D); #20. (B)

2001 AMC 12 Sample Problems:

1. The sum of two numbers is S. Suppose 3 is added to each number and then each of the resulting numbers is doubled. What is the sum of the final two numbers?

(A) 2S + 3 (B) 3S + 2 (C) 3S + 6 (D) 2S + 6 (E) 2S + 12

2. Let P(n) and S(n) denote the product and the sum, respectively, of the digits of the integer n. For example, P(23) = 6 and S(23) = 5. Suppose N is a two-digit number such that N = P(N) + S(N). What is the units digit of N?

(A) 2 (B) 3 (C) 6 (D) 8 (E) 9

9. Let f be a function satisfying $f(xy) = \frac{f(x)}{y}$ for all positive real numbers x and y. If f(500) = 3, find f(600).

(A) 1 (B) 2 (C) $\frac{5}{2}$ (D) 3 (E) $\frac{18}{5}$

14. Given the nine-sided regular polygon $A_1A_2A_3A_4A_5A_6A_7A_8A_9$, how many distinct equilateral triangles in the plane of the polygon have at least two vertices in the set $(A_1, A_2, \ldots, A_9)$?

(A) 30 (B) 36 (C) 6 (D) 66 (E) 72

15. An insect lives on the surface of a regular tetrahedron with edge of length 1. It wishes to travel on the surface of the tetrahedron from the midpoint of one edge to the midpoint of the opposite edge. What is the length of the shortest such trip? (Note: Two edges of a tetrahedron are opposite of they have no common endpoint.)

(A) $1\sqrt{3}$ (B) 1 (C) $\sqrt{2}$ (D) 3 (E) 2

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Answers
12: #1 (E); #2 (E); #9 (C); #14 (D); #15 (B)