Exam is next time:

1. Find the limit.

(a) $\lim_{x\to 1} (x+2)(x^2+1) = 6$

$(1+2)(1^2+1) =$
$(3)(2) = 6$

(b) $\lim_{x\to 2} \frac{1}{x-2}$ = Does not exist. (like $\frac{1}{x}$) (limit does not exist $x\to 0$)

$\lim_{x\to 2^+} \frac{1}{x-2} = \infty$ ($x>2$)

not equal so no limit

$\lim_{x\to 2^-} \frac{1}{x-2} = -\infty$ ($x<2$) $(x-2<0)$

(c) $\lim_{x\to -\infty} \frac{x}{x^3} = 0$

$\frac{x}{x^3} = \frac{1}{x^2}$ $\lim_{x\to -\infty} \frac{1}{x^2} = 0$

$x^2>0$ always
(when bottom power > top)

(d) originally $\lim_{x\to 1} \frac{x^2-2x-3}{1-x} =$ $-\infty$ (close) does not exist, correct

$1-x=0$ when $x\to 1$
but $x^2-2x-3 =$
$1^2-(2)-3 =$
$1-5 = -4$

Kind of dumb.

$x\to 1^+$ $1-x<0$ $\lim = +\infty$
$x\to 1^-$ $1-x>0$ so $\lim = -\infty$

(d) (better) $\lim_{x\to 1} \frac{x^2+2x-3}{1-x} = -4$ now both $\left(\begin{matrix}1^2+2-3=0 \\ 1-x=0\end{matrix}\ \text{"}\frac{0}{0}\text{"}\right)$

do algebra to simplify $x^2+2x-3 = (x+3)(x-1)$

$\frac{x^2+2x-3}{1-x} = \frac{(x+3)(x-1)}{(1-x)} = -(x+3).$ (negative of)

$\lim_{x\to 1} = \lim_{x\to 1} -(x+3) = -(1+3) = -4$

(e) $\lim_{t\to 0^+} \frac{\sin 4t}{3t} = \frac{4}{3}$

$\lim \frac{\sin 4t}{3t} = \lim_{t\to 0^+} \frac{1}{3}\left(\frac{\sin 4t}{t}\right)$
$= \frac{1}{3}(4)\left(\lim_{t\to 0^+} \frac{\sin t}{t}\right)$
$= \frac{1}{3}(4)(1) = \frac{4}{3}$

(f) $\lim_{x\to 1} \frac{2x^2-2}{\sqrt{x-1}}$ = does not exist (original)

when $x<1$ $\sqrt{x-1} = \sqrt{\text{negative number}}$
so $\lim_{x\to 1} \frac{2x^2-2}{\sqrt{x-1}}$ = does not exist.

New version $\lim_{x\to 1^+} \frac{2x^2-2}{\sqrt{x-1}}$ ("$\frac{0}{0}$")

($x>1$ so $\sqrt{x-1}$ is ok)

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do algebra rationalize denominator

$\frac{2x^2-2}{\sqrt{x-1}} \cdot \frac{\sqrt{x-1}}{\sqrt{x-1}} = \frac{(2x^2-2)\sqrt{x-1}}{(x-1)}$ $\left(\frac{(\sqrt{x-1})(\sqrt{x-1})}{x-1}\right)$

factor this

$\frac{2(x^2-1)\sqrt{x-1}}{(x-1)} = \frac{2(x-1)(x+1)(\sqrt{x-1})}{(x-1)}$

$= 2(x+1)(\sqrt{x-1})$

$\lim = \lim_{x\to 1} 2(x+1)(\sqrt{x-1}) = 2(1+1)\sqrt{1-1} = 2(2)(0) = 0$ ✓

$\lim_{x\to 1^+} \frac{2x^2-2}{\sqrt{x-1}} = 0$

(g) $\lim_{h\to 1} \frac{|h-1|}{h} =$ bottom not 0

$|x| = x$ when $x \geq 0$, $-x$ when $x<0$.

$\frac{\lim_{h\to 1} |h-1|}{\lim_{h\to 1} h} = \frac{|0|}{1} = 0$ ✓

(h) $\lim_{h\to 0} \frac{(2+h)^2-4}{h} = 4$ $\left(\frac{4-4}{0} \text{"}\frac{0}{0}\text{"}\right).$

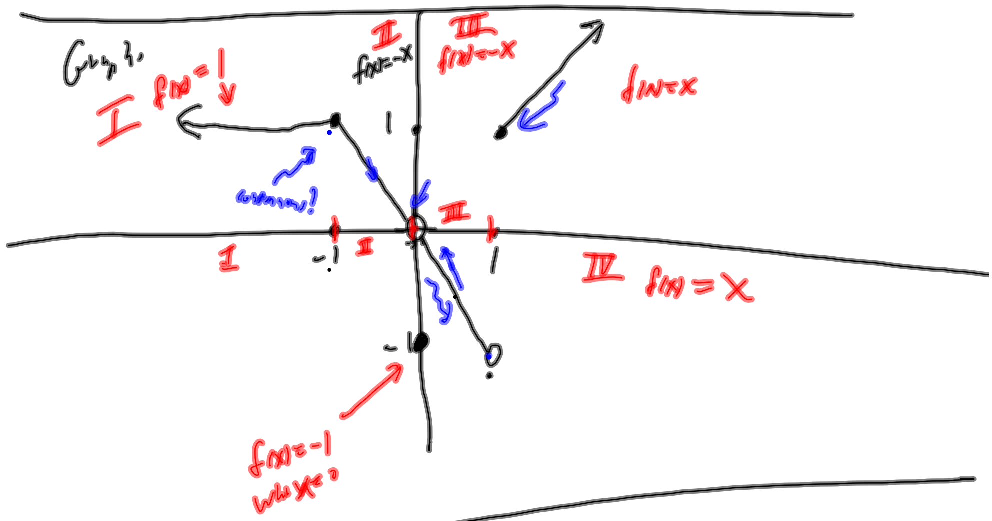
multiply top $\lim_{h\to 0} \frac{(4+4h+h^2)-4}{h}$

$\lim_{h\to 0} \frac{4h+h^2}{h} = \lim_{h\to 0} 4+h = 4+0 = 4$ ✓

2. Suppose f is a piecewise function:

$$f(x) = \begin{cases} 1 & \text{when } x \le -1 \quad \text{I} \\ -x & \text{when } -1 < x < 0 \quad \text{II} \\ -1 & x = 0 \leftarrow 1 \text{ point} \\ -x & \text{when } 0 < x < 1 \quad \text{III} \\ x & x \ge 1 \quad \text{IV} \end{cases}$$

Graph:



(ii) find limit or say does not exist.

(a) $\lim_{x\to 1^-} f(x) = -1$ (look at graph (III))

(b) $\lim_{x\to 1^+} f(x) = 1$ ($x \ge 1$, look at graph (IV))

(c) $\lim_{x\to 1} f(x)$ = does not exist ($1^+ \overset{\lim}{\neq} 1^-$)

(d) $\lim_{x\to 0^-} f(x) = 0$. (e) $\lim_{x\to 0^+} f(x) = 0$ (look at graph!) (f) $\lim_{x\to 0} f(x) = 0$ (both $= 0$, $0^+ = 0^-$)

(iii) (a) Is f continuous at $x=-1$? Yes no hole or jump.

(b) is f continuous at $x=0$? No. Hole at $x=0$

or $\lim_{x\to 0} f(x) = 0 \neq -1 = f(0)$

(3) Use the limit definition to calculate the derivative at $f(x) = \frac{x+1}{x-1}$.

either use the $x_0 \to x$ or $h \to 0$ version

$$f'(x) = \lim_{x_0 \to x} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x_0 \to x} \frac{\left(\frac{x+1}{x-1}\right) - \left(\frac{x_0+1}{x_0-1}\right)}{x - x_0}$$

do algebra to simplify

get common denom on top

$$\frac{\left(\frac{x+1}{x-1}\right) - \left(\frac{x_0+1}{x_0-1}\right)}{(x-x_0)} = \frac{\left(\frac{x+1}{x-1}\right)\cdot\left(\frac{x_0-1}{x_0-1}\right) - \left(\frac{x_0+1}{x_0-1}\right)\left(\frac{x-1}{x-1}\right)}{(x-x_0)}$$

$$= \frac{\left(\frac{(x+1)(x_0-1) - (x_0+1)(x-1)}{(x-1)(x_0-1)}\right)}{(x-x_0)}$$

$$= \frac{\frac{(xx_0 + x_0 - x - 1) - (x_0x + x - x_0 - 1)}{(x-1)(x_0-1)}}{(x-x_0)}$$

$$= \frac{\frac{x_0 - x - x + x_0}{(x-1)(x_0-1)}}{(x-x_0)} = \frac{\left(\frac{2x_0 - 2x}{(x-1)(x_0-1)}\right)}{(x-x_0)}$$

$$= \frac{\left(\frac{-2(x-x_0)}{(x-1)(x_0-1)}\right)}{x-x_0} = \frac{-2}{(x-1)(x_0-1)}$$

$$\lim_{x_0 \to x} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x_0 \to x} \frac{-2}{(x-1)(x_0-1)} = \frac{-2}{(x-1)(x-1)} = \frac{-2}{(x-1)^2}$$

Check with quotient rule. $\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

$$\frac{x+1}{x-1} \longrightarrow \frac{1(x-1) - (x+1)(1)}{(x-1)^2} = \frac{x - 1 - x - 1}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

✓ checks out

4. position $m(t) = t^2 - 2t$

(a) how long before velocity $= 2$?

velocity $= m'(t) = 2t - 2$

$2t - 2 = 2 \Rightarrow 2t = 4$

$t = 2.$

(b) How far is it when velocity $= 2.$

from (a) $t = 2$

$m(2) = 2^2 - 2(2) = 0.$

5. $y = x^3 + 2x + 2.$ $(= f(x))$

(a) what is equation for tangent line at $x = 0$?

$(y = 2 = 0^3 + 2(0) + 2.)$

(Secant line)

slope = $f'(x)$. $f'(0) =$

$f'(x) = 3x^2 + 2$ $\quad f'(0) = 3(0)^2 + 2 = 2.$

line $y = mx + b$ $\quad m$ - slope $\quad y = 2 \quad x = 0$

$2 = 2(0) + b \Rightarrow b = 2$

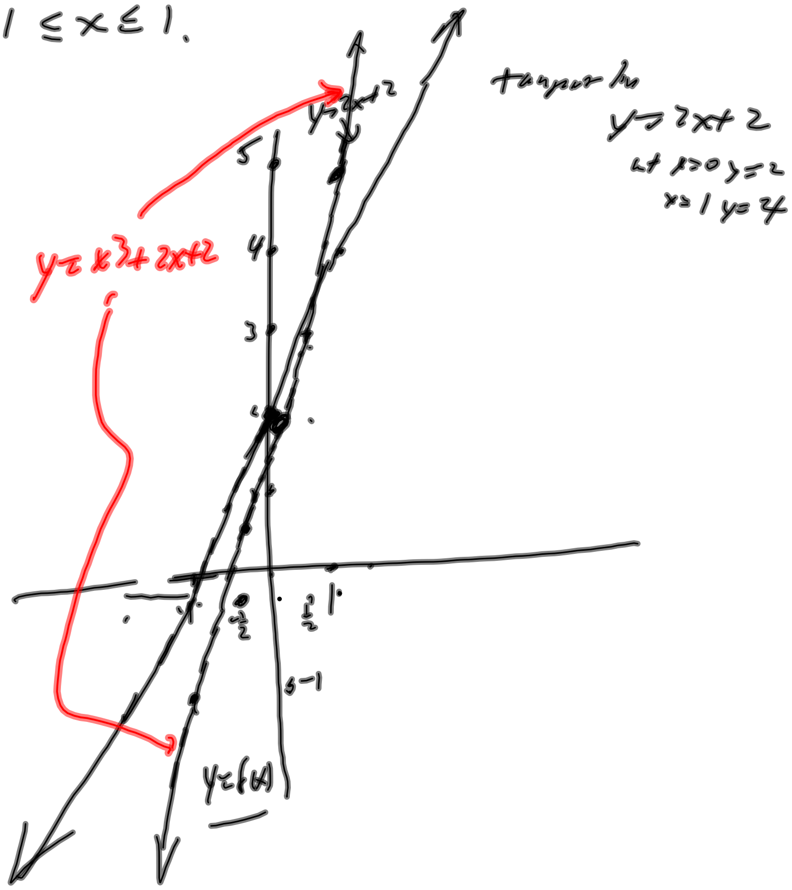
$y = 2x + 2.$ tangent line

(b) graph $y = f(x)$ and tangent line at $x = 0$

on $-1 \le x \le 1.$

$y = x^3 + 2x + 2$

x	y
0	2
1	5
-1	-1
$-\frac{1}{2}$	$(-\frac{1}{2})^3 + 2(-\frac{1}{2}) + 2 = -\frac{1}{8} - 1 + 2 = 1 - \frac{1}{8} = \frac{7}{8}$
$\frac{1}{2}$	$(\frac{1}{2})^3 + 2(\frac{1}{2}) + 2 = \frac{1}{8} + 1 + 2 = 3\frac{1}{8}$



(6) Find derivative

(a) $f(x) = x^3$

$f'(x) = 3x^2$

(b) $f(x) = |x|$

$f(x) = x$ when $x \geq 0$, $-x$ when $x < 0$

$f'(x) = 1$ when $x \geq 0$, -1 when $x < 0$

(c) $f(x) = \sqrt{x} - 2x + 1$

$f'(x) = \frac{1}{2\sqrt{x}} - 2$

$f(x) = x^{\frac{1}{2}} - 2x + 1$

$f'(x) = \frac{1}{2}(x^{-\frac{1}{2}}) - 2$

$= \frac{1}{2}\left(\frac{1}{\sqrt{x}}\right) - 2 = \frac{1}{2\sqrt{x}} - 2.$

(d) $f(x) = 10^{100}$ constant

$f'(x) = 0$

.... derivatives known ...

(e) $f(x) = \left(\frac{\sin x}{\sqrt{x}}\right)(1 + \sec x)$

product rule & trig.

quotient rule $\frac{\sin x}{\sqrt{x}}$

$\frac{\cos x(\sqrt{x}) - \sin x\left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x})^2}$ => done w/ 1st

$f'(x) = \frac{\cos x(\sqrt{x}) - \sin x \frac{1}{2\sqrt{x}}}{(\sqrt{x})^2}$

$f'(x) = \left(\frac{\cos x(\sqrt{x}) \cdot \sin x\left(\frac{1}{2\sqrt{x}}\right)}{x}\right)(1 + \sec x) + \left(\frac{\sin x}{\sqrt{x}}\right)(\sec x \tan x)$ ✓

ugly but right

(f) $f(x) = \sin x \sin x + \cos x \cos x$

$f'(x) = (\cos x)(\sin x) + \sin x(\cos x) + \cos x(\cos x) + \sin x(-\sin x)$

$2(\cos x \sin x) + \cos^2 x - \sin^2 x$ = done but ...

$(= \sin 2x + \cos 2x$ trig identity$)$

(g) $f(x) = \tan(x^2 - x)\, x$

chain rule

$f'(x) = \left(\sec^2(x^2 - x) \cdot (2x - 1)\right) \cdot x + \tan(x^2 - x)(1)$

$+ \tan(x^2 - x)$

(h) $f(x) = (\cos 2x)^{\frac{1}{2}}$ chain rule 3 times

$= \left(\frac{1}{2}(\cos 2x)^{\frac{1}{2} - 1}\right) \cdot (-\sin 2x) \cdot (2)$

$= \frac{-\sin 2x}{\sqrt{\cos 2x}}.$

(i) $f(x) = \frac{\cos^2 x + \sin(2x) + x}{\sin^3(x)}$ quotient rule

chain rule, chain

$f'(x) = \frac{(2\cos x(-\sin x) + (\cos 2x)(2) + 1)(\sin^3 x) - (\cos^2 x + \sin 2x + x)(3\sin^2 x)(\cos x)}{(\sin^3 x)^2\ (= \sin^6 x)}$

$((3\sin^2 x)(\cos x))$

(j) $f(x) = \frac{(x^2)(\sin x)}{(x^{2/3} - x)(\cot x)}$ quotient rule

$f'(x) = \frac{(2x \sin x + x^2(\cos x)) \cdot (x^{2/3} - x)(\cot x) - x^2 \sin x\left[\left(\frac{2}{3}x^{-\frac{1}{3}} - 1\right)(\cot x) + \ldots \ldots + (x^{2/3} - x)(-\csc^2 x)\right]}{((x^{2/3} - x)(\cot x))^2}$

yikes!

7. Implicit differentiation (derivative of whole equation then solve for dy/dx.)

$x^2 + y^3 = \sin x$ derivative

$$\frac{d}{dx}(x^2 + y^3) = \frac{d}{dx}(\sin x)$$

$$2x + 3y^2\left(\frac{dy}{dx}\right) = \cos x$$

$$3y^2\left(\frac{dy}{dx}\right) = \cos x - 2x$$

$$\frac{dy}{dx} = \frac{\cos x - 2x}{3y^2}.$$

$$\frac{d}{dx}(y^3) =$$

$$\frac{d(y^3)}{dy} \cdot \frac{dy}{dx}$$

$$3y^2\left(\frac{dy}{dx}\right)$$